fundamental. It may not be inappropriate, therefore, to devote the remainder of this address to an elementary exposition of the meaning of these two quantities, not with the thought of adding anything essentially new to the theory, but in the hope of providing a convenient starting point from which some of us who are not already familiar with the subject of statistics may begin our study of its modern developments.

3. The Central Problem of Statistics. Correlation between Two Functions, $x(i)$, $y(i)$.—In the problem of correlation, what is sought for is some measure of agreement or disagreement between *two series of paired quantities*, $x_1, x_2, x_3, \cdots x_n$ and $y_1, y_2, y_3, \cdots y_n$, where the pair x_1, y_1 are supposed to belong to an "individual" numbered 1, the pair x_2, y_2 to an "individual" numbered 2, and so on; the total number of individuals, n, being called the total "population."

For example, x and y may be the height and weight of individual men; or x may be the rainfall and y the price of wheat for individual years; and so on.

In order to state the problem geometrically, we may plot the x's and y's as ordinates against the individual-numbers i as abscissas, as in Fig. 1; what we seek is then some measure of agreement or disagreement between the *two curves or functions*

$$x = f(i), \qquad y = \varphi(i)$$

over a range of values $a \leqq i \leqq b$ (the function being here defined for only a finite number of values of i).

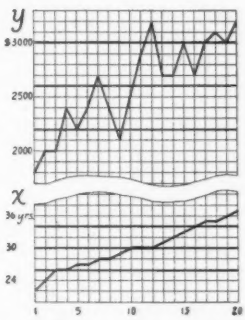

FIG. 1. x = age, y = income, of a certain group of men ($n = 20$). Means: $\bar{x} = 30$ years, $\bar{y} = 2600$ dollars. Standard deviations: $\sigma_x = 4.025$ years, $\sigma_y = 417.1$ dollars. Correlation coefficient: $r = .855$.

4. Notation: x, y; ξ, η; X, Y.—In order to reduce these two curves to a comparable basis, it is convenient to take two preliminary steps, one concerning the base-line, or origin, the other concerning the scale, or unit of measurement.

As to the *base-line*, we agree to refer each of the given series to the (arithmetical) mean of that series as origin, the means being given by

$$\bar{x} = \frac{1}{n}\sum (x_i), \qquad \bar{y} = \frac{1}{n}\sum (y_i).$$

In other words, instead of plotting the given x's and y's, we plot the "deviations from the means," ξ and η, where

$$\xi_i = x_i - \bar{x}, \qquad \eta_i = y_i - \bar{y}.$$

As to the *scale*, we agree to take as the unit of measurement for each series the "standard deviation" of that series, namely

$$\sigma_x = \sqrt{\frac{1}{n}\sum (\xi_i^2)}, \qquad \sigma_y = \sqrt{\frac{1}{n}\sum (\eta_i^2)},$$

where the "standard deviation," σ_x (or σ_y), may be interpreted as the radius of gyration of the x-curve (or y-curve) about a horizontal axis through its mean height. In other words, instead of plotting the x's and y's or the ξ's and η's in their original units (kilograms, dollars, degrees, or what not), we plot the "ratios" X and Y (as in Fig. 2), where

$$X_i = \xi_i/\sigma_x, \qquad Y_i = \eta_i/\sigma_y.$$

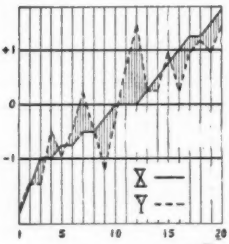

FIG. 2. $X = \dfrac{x - \bar{x}}{\sigma_x}$, $Y = \dfrac{y - \bar{y}}{\sigma_y}$.

These X's and Y's will be pure numbers, independent of the original units, and having the following properties:

$$\frac{1}{n}\sum (X_i) = 0, \qquad \frac{1}{n}\sum (Y_i) = 0;$$

$$\frac{1}{n}\sum (X_i^2) = 1, \qquad \frac{1}{n}\sum (Y_i^2) = 1.$$

That is, the arithmetical mean of X and the arithmetical mean of Y are both zero, while their standard deviations (or radii of gyration) are both unity.

For the purpose of studying correlation, the reduced curves formed by plotting X_i and Y_i against i will be more convenient than the original curves formed by plotting x_i and y_i against i; and any criterion of agreement or disagreement between the X's and Y's may be accepted as a valid criterion of agreement or disagreement between the x's and y's.

Note.—The term "standard deviation" was introduced by Pearson in 1894.

It should be noted that in case x (or y) is a constant, X (or Y) will be indeterminate. This case, which presents no special interest, will be excluded from consideration in what follows.

5. The Correlation Coefficient r. Definition and Properties.—The most widely accepted measure of correlation between two given series or curves x and y, is the Pearsonian (or "product-moment") *coefficient of correlation*, r, which may be defined in terms of the reduced quantities X and Y by the simple formula

$$r = \frac{1}{n}\sum (X_i Y_i).$$

That is, r is the mean product of corresponding pairs of values of X and Y. By aid of the simple transformations

$$r = 1 - \frac{1}{2}\frac{1}{n}\sum [(X_i - Y_i)^2], \qquad r = -1 + \frac{1}{2}\frac{1}{n}\sum [(X_i + Y_i)^2],$$

it is easily shown that r cannot exceed 1 in absolute value. The case $r = +1$, called the case of *perfect positive correlation*, will occur when and only when the X and Y curves coincide. The case $r = -1$, called *perfect negative correlation*,

will occur when and only when the Y curve is the reflection of the X curve in the axis of i. Either case will occur when and only when the η's are directly proportional to the ξ's; or, what amounts to the same thing, when and only when the original y's and x's are connected by a linear equation

$$\frac{y_i - \bar{y}}{\sigma_y} = \pm \frac{x_i - \bar{x}}{\sigma_x}.$$

We see, therefore, that in the Pearsonian sense, *perfect correlation (positive or negative) between two sets of quantities x and y means nothing more nor less than the existence of a linear algebraic equation connecting those quantities.* Indeed, a better name for the coefficient of correlation might be the "coefficient of linear relationship."

In general, the given sets of values will not be linearly related, and the value of r will be less than 1.

6. Relation of r to the Method of Least Squares.—The significance of the coefficient of correlation, r, may be brought out further by the following considerations. In the expression

$$r = 1 - \frac{1}{2}\frac{1}{n}\sum [(X_i - Y_i)^2],$$

the quantity

$$\Delta = \frac{1}{2}\frac{1}{n}\sum [(X_i - Y_i)^2]$$

may be regarded as a measure of the *total discrepancy*, in the sense of the method of least squares, between the curves X and Y. Clearly, when the discrepancy Δ is zero, or the curves coincide, the correlation r is perfect ($r = 1$); and as the discrepancy increases the correlation decreases. Hence r is seen to be a suitable measure of the degree of approach to coincidence of the two curves X and Y.

We note that as r varies from $+1$ to -1, Δ will vary from 0 to 2, the value $r = 0$ corresponding to $\Delta = 1$.

A further connection with the method of least squares will be noted below (§ 9).

7. Equivalent Formulas for r. The Case of a Continuous Variable.—Equivalent and more familiar formulas for r are

$$r = \frac{\sum(\xi\eta)}{n\sigma_x\sigma_y}, \quad \text{or} \quad r = \frac{\sum(\xi\eta)}{\sqrt{\sum(\xi^2)}\sqrt{\sum(\eta^2)}}.$$

The importance of the product-factor $\Sigma(\xi\eta)$ was evident in the work of Bravais in 1846, and was re-discovered and applied by Galton in 1886–1888; but the complete expression for r, in the form just stated, was first given by Pearson in 1896. [Compare also L. March (1905).]

For purposes of numerical computation, the following formula is to be preferred in practice (J. A. Harris, *Amer. Naturalist*, Vol. 44, p. 693–699, 1910; L. L. Thurstone, *Psychological Bulletin*, Vol. 14, pp. 28–32, 1917):

$$r = \frac{\frac{1}{n}\sum(xy) - \bar{x}\bar{y}}{\sqrt{\frac{1}{n}\sum(x^2) - \bar{x}^2}\sqrt{\frac{1}{n}\sum(y^2) - \bar{y}^2}},$$

where, in the denominator,

$$\sigma_x = \sqrt{\frac{1}{n}\sum(x^2) - \bar{x}^2} \quad \text{and} \quad \sigma_y = \sqrt{\frac{1}{n}\sum(y^2) - \bar{y}^2}$$

are the standard deviations of the x's and y's.

We note in passing that if the n individual-numbers, i, are replaced by a continuous variable, t, the formulas above become modified as follows (the summations being replaced by integrations): Given $x = f(t)$ and $y = \varphi(t)$ for the range $a \leqq t \leqq b$. Let

$$\bar{x} = \frac{1}{b-a}\int_a^b x\,dt, \qquad \bar{y} = \frac{1}{b-a}\int_a^b y\,dt; \qquad \xi = x - \bar{x}, \qquad \eta = y - \bar{y};$$

$$\sigma_x^2 = \frac{1}{b-a}\int_a^b \xi^2\,dt, \qquad \sigma_y^2 = \frac{1}{b-a}\int_a^b \eta^2\,dt; \qquad X = \frac{\xi}{\sigma_x}, \qquad Y = \frac{\eta}{\sigma_y}.$$

Then

$$\frac{1}{b-a}\int_a^b X\,dt = 0, \quad \frac{1}{b-a}\int_a^b Y\,dt = 0; \quad \frac{1}{b-a}\int_a^b X^2\,dt = 1, \quad \frac{1}{b-a}\int_a^b Y^2\,dt = 1;$$

and

$$r = \frac{1}{b-a}\int_a^b XY\,dt = \frac{\int_a^b \xi\eta\,dt}{(b-a)\sigma_x\sigma_y} = \frac{\frac{1}{b-a}\int_a^b xy\,dt - \bar{x}\bar{y}}{\sqrt{\frac{1}{b-a}\int_a^b x^2\,dt - \bar{x}^2}\sqrt{\frac{1}{b-a}\int_a^b y^2\,dt - \bar{y}^2}}.$$

8. The Correlation Graph, or Scatter-Diagram.—Up to this time we have thought of the values of x_i and y_i as functions of i, and have plotted them as two separate curves on the axis of i as a base. It is often more convenient to think of y as a function of x, and to plot the graph of y against x in the ordinary way. The result will not, however, be an ordinary graph, since y will not, in general, be a single-valued function of x. To any value of x may correspond many values of y, any one of which may be repeated more than once. A typical graph of y as a function of x will therefore appear as in Fig. 3, called a *correlation graph* (Galton, 1888), in which every dot (x, y) represents a pair of values of x and y belonging to some individual, and the total number of dots is equal to the total "population," n. (Multiple dots would be indicated in the figure by clusters.)

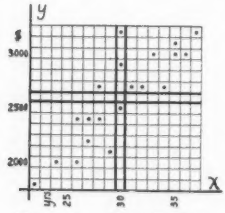

FIG. 3. Correlation graph.

In practice, such a graph is usually divided into squares of suitable size, and the number of dots falling within each square is indicated by a numeral written within that square. This gives a "correlation table" for x and y. Or, each such numeral may be replaced by a z-ordinate of corresponding height, erected at the middle of the square, and a sort of tent-cloth spread over the tops of these ordinates. This gives a "correlation surface," the height of which at any point of the x, y plane shows the density of the distribution of the dots at that point. For our present purpose, however, the simple correlation graph or "scatter-diagram," is all that we shall need.

9. Regression Lines and Coefficients of Regression.—Let us now seek a linear expression that shall give us most accurately the value of y corresponding to any given x.

In the special case when $r = 1$ we know (§ 5) that y is simply a linear function of x, and the equation

$$y - \bar{y} = \frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad \text{or} \qquad Y = X$$

will give y exactly in terms of x (or Y in terms of X). In this special case all the dots in the correlation graph will lie on this line. In the general case (r not equal to 1), we may write an approximate equation

$$y' - \bar{y} = \lambda \frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad \text{or} \qquad Y' = \lambda X$$

and seek to determine the arbitrary factor λ so that the values of y' (or Y') obtained from this equation shall be equal as nearly as possible to the true values of y (or Y). That is, we seek to determine λ so that the "least-squares" total error,

$$e_y^2 = \frac{1}{n}\sum[(y' - y)^2] \qquad \text{or} \qquad E_y^2 = \frac{e_y^2}{\sigma_y^2} = \frac{1}{n}\sum[(Y' - Y)^2]$$

shall be a minimum. An easy transformation (using the simpler notation) gives

$$E_y^2 = 1 - r^2 + (\lambda - r)^2,$$

which will clearly be a minimum when $\lambda = r$. Hence the "best" straight line to give y in terms of x will be

$$y' - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad \text{or} \qquad Y' = rX,$$

while the amount of total error involved in using y' for y (or Y' for Y) is

$$e_y^2 = \sigma_y^2(1 - r^2) \qquad \text{or} \qquad E_y^2 = 1 - r^2.$$

Clearly, if $r = 1$, this error is zero; and if $r = 0$, the error takes its maximum value. Hence again we see that the value of r is a suitable criterion of the approach to linear relationship of the variables x and y.

The straight line just obtained is called the *line of regression* of y on x, and the factor $r(\sigma_y/\sigma_x)$ is called the *coefficient of regression of y on x.*

Similarly, the straight line

$$x' - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y}) \qquad \text{or} \qquad X' = rY$$

will be the "best" straight line for giving x in terms of y, the "total error" being

$$e_x^2 = \sigma_x^2(1 - r^2) \qquad \text{or} \qquad E_x^2 = 1 - r^2.$$

This line is called the *line of regression of x on y*, and the factor $r(\sigma_x/\sigma_y)$ is called the *coefficient of regression of x on y.*

The two lines of regression will not coincide unless $r = \pm 1$.

It will be observed that *the coefficient of correlation, r, is the geometrical mean of the two coefficients of regression.* This fact is the starting point for the theory of "partial correlation coefficients" for any number of variables (Yule, 1897 and 1907), into which we cannot enter here.[1]

10. Curves of the Means, or Regression Curves.—The correlation graph, with its scattered dots, may, for some purposes, be simplified as follows.

Thinking of y as a function of x, let us replace each column by a representative dot located at the mean of that column. We thus obtain what is called the *curve of the means of the columns,* or the *regression curve of y on x* (Pearson, 1896).

Or again, thinking of x as a function of y, we may replace each row by a

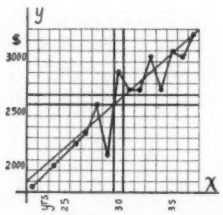

FIG. 4. Curve of the means of the columns, and line of regression of y on x. Coefficient of regression (y on x) = .886; correlation ratio (y on x) = η_{yx} = .941.

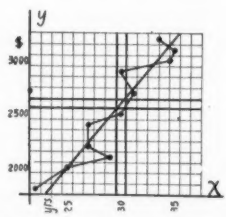

FIG. 5. Curve of the means of the rows, and line of regression of x on y. Coefficient of regression (x on y) = .825; correlation ratio (x on y) = η_{xy} = .916.

representative dot located at the mean of that row, thus obtaining the *curve of the means of the rows* or the *regression curve of x on y.* (See Figures 4 and 5).

If the curves of the means happen to be straight lines, we have the case of *linear regression,* which was the earliest, and for a time the only case considered (Galton, 1888; Pearson, 1895; Pearson, 1905). *In this case it can be readily shown that the curves of the means, or curves of regression, will coincide with the lines of regression defined in* § 9.

[1] An excellent example of the application of partial correlation coefficients in practical statistics may be found in W. F. Ogburn's "Analysis of the standards of living in the District of Columbia in 1916:" *Quart. Pub. Amer. Statistical Association,* vol. 16, June, 1919, pp. 374–389.

In order to prove this most clearly, for the case, say, of the column-means, let us adopt the following notation, expressed, for convenience, in terms of X and Y (§ 4):

X = the abscissa of any column
n_X = the number of dots in the column
Y_X = the ordinate of any dot in the column
$\overline{Y}_X$ = the mean of the column; that is,

$$\overline{Y}_X = \frac{1}{n_X} S_X(Y_X),$$

where S_X denotes summation from dot to dot within the column (X = constant).

These $\overline{Y}_X$'s are the ordinates of the curve of the means in question. If this curve is a straight line, then we must have

$$\overline{Y}_X = \mu + \lambda X,$$

where μ and λ are some constants. But, summing from dot to dot over the whole table,

$$\frac{1}{n}\sum(\overline{Y}_X) = \frac{1}{n}\sum(\mu) + \frac{1}{n}\sum(\lambda X)$$

or[1]

$$\frac{1}{n}\sum(Y) = \mu + \lambda\frac{1}{n}\sum(X),$$

whence

$$\mu = 0.$$

Again, multiplying each side by X, and summing as before,

$$\frac{1}{n}\sum(X\overline{Y}_X) = \lambda\frac{1}{n}\sum(X^2)$$

or[2]

$$\frac{1}{n}\sum(XY) = \lambda(1),$$

whence

$$\lambda = r.$$

Hence, if the curve is a straight line at all, it must be the line

$$\overline{Y}_X = rX$$

which is precisely the regression line of y on x.

[1] Thus, if S_A denotes summation from column to column, while S_X denotes summation from dot to dot within the column X, then

$$\frac{1}{n}\Sigma(\overline{Y}_X) = \frac{1}{n}S_A[S_X(\overline{Y}_X)] = \frac{1}{n}S_A[S_X(Y)] = \frac{1}{n}\Sigma(Y) = 0.$$

[2] Thus,

$$\frac{1}{n}\Sigma(X\overline{Y}_X) = \frac{1}{n}S_A[S_X(X\overline{Y}_X)] = \frac{1}{n}S_A[XS_X(\overline{Y}_X)] = \frac{1}{n}S_A[XS_X(Y)] = \frac{1}{n}S_A[S_X(XY)] + \frac{1}{n}\Sigma(XY) = r.$$

11. The Correlation Ratio, η_{yx}. Definition and Properties.—Whether the curve of the means of the columns is linear or not, the whole system of dots in a correlation graph may be regarded as forming a band of more or less irregular width along this curve; and the *narrower this band, the more nearly will y be a single-valued function of x.*

On the other hand, the dots may be regarded as forming a band along the curve of the means of the rows; and the narrower this band, the more nearly will x be a single-valued function of y.

Considerations of this sort led Pearson in 1905 to introduce two new measures of correlation, η_{yx} and η_{xy}, called the *correlation ratio of y on x* and the *correlation ratio of x on y* respectively. If one speaks simply of *the* correlation ratio, η, one usually means the correlation ratio of y on x. The definition and properties of this quantity may be explained as follows.

In order to secure a measure of the width of the band of dots about the curve of the means of the columns, we begin by letting

d_X = the standard deviation of a column, or the radius of gyration of the column about its mean; that is,

$$d_X = \sqrt{\frac{1}{n_X} S_X(Y_X - \overline{Y}_X)^2},$$

where S_X denotes summation from dot to dot within the column (§ 10). These d_X's will measure the "scatter" of the dots in each column about the mean of that column.

What we then need, as a measure of the total "scatter" of the dots about the curve of the means, is some kind of average of the d_X's for all the columns.

The most obvious average to take for this purpose would be either the *mean* of the d_X's, namely,

$$D' = \frac{1}{A} S_A(d_X),$$

where S_A denotes summation from column to column, and A is the number of columns, or else the *standard deviation* of the d_X's, namely,

$$D'' = \sqrt{\frac{1}{A} S_A(d_X^2)}.$$

It is found more convenient, however, to take a sort of *pseudo-standard deviation*, namely,

$$D = \sqrt{\frac{1}{n} S_A(n_X d_X^2)}$$

in which each d_X^2 is "weighted" by the number of dots in that column, and the sum divided, consequently, by the total number of dots, n, instead of by the number of columns, A.[1]

[1] Here D' is the mean height and D'' the radius of gyration of the "scedastic curve" formed by plotting d_X against X. Both these averages were considered by Pearson (1905, p. 10), but immediately abandoned in favor of D.

This quantity D may be transformed without difficulty as follows:[1]

$$D^2 = \frac{1}{n} S_A[S_X(Y_X - \bar{Y}_X)^2] = 1 - \frac{1}{n} S_A(n_X \bar{Y}_X{}^2).$$

The *correlation ratio of y on x*, is then defined as

$$\eta_{yx} = \sqrt{\frac{1}{n} S_A(n_X \bar{Y}_X{}^2)}$$

where S_A denotes, as before, summation from column to column.

This quantity η_{yx} clearly lies between 0 and 1; and the relation $\eta_{yx}{}^2 = 1 - D^2$ shows that the case $\eta_{yx} = 1$ *will occur when and only when the band of dots narrows down into coincidence with the curve of the means of the columns; that is, when and only when y is a single-valued function of x.* The narrower the band, the more nearly will η_{yx} approach 1.

In terms of the original variables, x and y, the definition becomes

$$\eta_{yx} = \frac{\sqrt{\frac{1}{n} S_A(n_x \bar{y}_x{}^2) - \bar{y}^2}}{\sqrt{\frac{1}{n} \Sigma(y^2) - \bar{y}^2}}$$

which is the most convenient form for computation.

The *correlation ratio of x on y*, namely η_{xy}, is obtained by simply interchanging x and y and replacing S_A by S_B (to denote summation from row to row).

It should be especially noted that the significance of the two correlation ratios is concerned with the single-valuedness of the connection between x and y, rather than with any particular form of connecting equation. Indeed, they might well be called the "coefficients of single-valued relationship."

For the case of more than two variables, definitions of "partial" and "multiple" correlation ratios have been given by Pearson (1915).

12. Relation between η_{yx} and r.—An important relation between the correlation ratio, η_{yx}, and the correlation coefficient, r, may be brought out by a further study of the curve of the means of the columns (the regression curve of y on x).

If the curve of the means happens to be a straight line, we have the case of *linear regression* (§ 10).

---

[1] Thus:

$$D^2 = \frac{1}{n} S_A[S_X(Y_X - \bar{Y}_X)^2]$$

$$= \frac{1}{n} S_A[S_X(Y_X{}^2)] - 2\frac{1}{n} S_A[\bar{Y}_X S_X(Y_X)] + \frac{1}{n} S_A[S_X(\bar{Y}_X{}^2)]$$

$$= \frac{1}{n} \Sigma(Y^2) - 2\frac{1}{n} S_A(\bar{Y}_X n_X \bar{Y}_X) + \frac{1}{n} S_A(n_X \bar{Y}_X{}^2)$$

$$= 1 - \frac{1}{n} S_A(n_X \bar{Y}_X{}^2).$$

If the curve of the means is not a straight line, we may seek the straight line which fits the curve with a minimum total discrepancy.

Let $Y' = \mu + \lambda X$ be the required straight line, where μ and λ are to be determined so as to make the total discrepancy between the line and the curve a minimum.

As the expression for the total discrepancy, it would be most natural to take the "least-squares" error

$$\frac{1}{A} S_A[(\mu + \lambda X - \overline{Y}_X)^2],$$

where S_A denotes summation from column to column, and A = the number of columns. It proves more convenient, however, to take a "pseudo-least-squares" expression

$$E^2 = \frac{1}{n} S_A[n_X(\mu + \lambda X - \overline{Y}_X)^2],$$

in which the square of the difference in each column is "weighted" by the number of dots in that column before summing. This quantity E^2 may be transformed, by a straightforward process,[1] into

$$E^2 = \eta_{yx}^2 - r^2 + \mu^2 + (\lambda - r)^2,$$

where η_{yx} has the value defined above (§ 11).

Evidently, to make E a minimum we must put $\mu = 0$ and $\lambda = r$. Hence the line which best fits the curve of the means of the columns (in the pseudo-least-squares sense) is simply the line of regression (§ 9):

$$Y' = rX \qquad \text{or} \qquad y' - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}).$$

The total discrepancy between the line and the curve is

$$E^2 = \eta_{yx}^2 - r^2 \qquad \text{or} \qquad e^2 = \sigma_y^2(\eta_{yx}^2 - r^2).$$

From this last equation we see that *the correlation ratio, η_{yx}, will be equal to the correlation coefficient, r, when and only when the regression of y on x is linear*; that is, when and only when the curve of the means of the columns reduces to a straight line.

---

[1] Noting that

$$\frac{1}{n} S_A(n_X X) = \frac{1}{n} \Sigma(X) = 0,$$

$$\frac{1}{n} S_A(n_X X^2) = \frac{1}{n} \Sigma(X^2) = 1,$$

$$\frac{1}{n} S_A(n_X \overline{Y}_X) = \frac{1}{n} S_A[S_X(Y_X)] = \frac{1}{n} \Sigma(Y) = 0,$$

and

$$\frac{1}{n} S_A(X n_X \overline{Y}_X) = \frac{1}{n} S_A[X S_X(Y_X)] = \frac{1}{n} S_A[S_X(XY_X) = \frac{1}{n} \Sigma(XY) = r.$$

Similarly, $\eta_{xy} = r$ when and only when the regression of x on y is linear.

Thus, in the case of linear regression, it makes no difference whether we use the correlation ratio or the correlation coefficient. In the case of non-linear regression, the correlation ratio is usually to be preferred.

It should be noted, in this connection, that one of the curves of the means may be a straight line and the other not, so that we may have "linear regression" for y on x and not for x on y.

13. Conclusion. Remarks on the Probable Error.—We have thus completed our sketch of the elementary theory of the correlation coefficient and the correlation ratio. Space does not permit any account of other methods of measuring correlation, such as the method of contingency, the method of correlation by ranks (including Spearman's "Foot-Rule"), the method of four-fold division, etc., which have been devised for special purposes (compare Pearson, 1907; Ritchie-Scott, 1918). Nor can we discuss the precautions that are necessary in practical computation on account of "corrections for grouping" in the correlation table.

At least a word must, however, be said in regard to the question of *probable errors*. The probable error of the correlation coefficient, r, is usually given as

$$.67449\,\frac{1-r^2}{\sqrt{n}}$$

and that of the correlation ratio, η_{yx}, as roughly,

$$.67449\,\frac{1-\eta_{yx}^2}{\sqrt{n}},$$

while the probable error of the quantity $\zeta = \eta_{yx}^2 - r^2$ (used as a test for linearity of regression) is, still more roughly,

$$.67449\left(2\,\frac{\sqrt{\zeta}}{\sqrt{n}}\right);$$

and we are continually warned that *no confidence can be placed in any of these quantities unless their value is three or four times their probable error.*

Now what is the real significance of these probable errors? They have required some extraordinarily intricate mathematics for their determination; it is hardly possible to explain their essential nature in a few words. This much, can, however, be said. The question is primarily a question of what is technically known as the "errors of random sampling." Unless the statistical material with which we are dealing is a "sample" of a larger statistical "population," no question of "probable error" will arise.

The simplest case may perhaps be stated as follows: Suppose a correlation table exists, with a population N, and an unknown correlation coefficient, r; and suppose a sample population, n, drawn from N, is found to have a correlation coefficient r_1. Another sample of the same size would doubtless have a different

coefficient, r_2. Suppose the total number of samples of this size that can be drawn from N is m, and let

$$r_1, r_2, r_3, \cdots r_m$$

be the correlation coefficients for these m samples. Further, let $\bar{r}$ be the mean of these r's, which we may take to be equal to the true value r, and let ρ be their standard deviation:

$$\rho = \sqrt{\frac{1}{m} \sum (r - \bar{r})^2}.$$

Then *if the r's are assumed to be distributed according to the normal law of error*, it can be shown that half of them will lie between $\bar{r} + 0.6745\rho$ and $\bar{r} - 0.6745\rho$, and the other half outside those limits. When we say, then, that an observed value, r_k, has a "probable error" of $p = .6745\rho$, or that the true value of r is as likely to lie within as without the limits

$$r_k + p \qquad \text{and} \qquad r_k - p,$$

we mean simply that *if we actually tested all the mathematically possible samples, half the results would lie within these limits and half without; the important point being that we do not know to which half our particular sample may belong.* Any statement as to the size of a probable error is always a statement of our ignorance; but even this ignorance may give wise information, for while all ignorance is deplorable, some ignorance is more deplorable than others. As a general rule, the smaller the size of the sample, the greater the depth of our ignorance.

14. References.—The full titles of the articles referred to above by date only, are here appended. The best bibliography of the original memoirs—to which every serious student of statistics must eventually refer—is given in Yule's *Introduction to the Theory of Statistics*, 3d edit., 1916.

A. Bravais, 1846. Sur les probabilités des erreurs de situation d'un point. *Mémoires présentés par divers savants à l'Académie royale des sciences de l'Institut de France; Sciences mathématiques et physiques*, Vol. 9, p. 255–332.

F. Galton, 1886. Family likeness in stature, with an appendix by J. D. Hamilton Dickson. *Proc. Royal Society of London*, Vol. 40, p. 42–73.

F. Galton, 1888. Co-relations and their measurement, chiefly from anthropometric data. *Proc. Royal Society of London*, Vol. 45, p. 135–145.

K. Pearson, 1894. Contributions to the mathematical theory of evolution. [I. Dissection of Compound Normal Frequency Curves.] *Phil. Trans. Roy. Soc. London*, Ser. A, Vol. 185, p. 71–110.

K. Pearson, 1895. Contributions to the mathematical theory of evolution. II. Skew variation in homogeneous material. *Phil. Trans. Roy. Soc. London*, Ser. A, Vol. 186, p. 343–414.

K. Pearson, 1896. Contributions to the mathematical theory of evolution. III. Regression, Heredity, and Panmixia. *Phil. Trans. Roy. Soc. London*, Ser. A, Vol. 187, p. 253–318.

G. U. Yule, 1897. On the theory of correlation. *Journ. Roy. Statistical Soc. London*, Vol. 60, p. 812–854.

K. Pearson, 1905. Mathematical contributions to the theory of evolution. XIV. On the general theory of skew correlation and non-linear regression. *Drapers' Company Research Memoirs, Biometric Series, II.* London, Dulau and Co., 1905 (54 pages).

L. March, 1905. Comparaison numérique de courbes statistiques. *Journ. de la Societé de Statistique de Paris*, Vol. 46, p. 255–277; 306–307.

K. PEARSON, 1907. Mathematical contributions to the theory of evolution. XVI. On further methods of determining correlation. *Drapers' Company Research Memoirs, Biometric Series, IV.* London, Dulau and Co., 1907. (39 pages.)

G. U. YULE, 1907. On the theory of correlation for any number of variables, treated by a new system of notation. *Proc. Roy. Soc. London*, Ser. A, Vol. 79, p. 182–193.

K. PEARSON, 1915. On the partial correlation ratio. *Proc. Roy. Soc. London*, Ser. A, Vol. 91, p. 492–498.

A. RITCHIE-SCOTT, 1918. The correlation coefficient of a polychoric table. *Biometrika*, Vol. 12, p. 93–133.

MEETING OF THE MINNESOTA SECTION.

The annual spring meeting of the Minnesota Section was held on Saturday, May 31, 1919, at Science Hall, Hamline University. There were thirty-one present including the following members of the Association: R. M. Barton, W. O. Beal, Edla G. Berger, W. H. Bussey, H. H. Dalaker, C. H. Gingrich, R. A. Johnson, Sister Mary John (institutional representative), R. M. Mathews, C. A. V. Peterson, Jessie G. Quigley, W. D. Reeve, Ella A. M. Thorp, C. H. Yeaton.

The program consisted of chairman's remarks by C. H. Gingrich, Carleton College, a paper by W. F. Swann, of the department of physics, University of Minnesota, a book review by W. O. Beal, of the department of astronomy, University of Minnesota, a memorial tribute to Father William Earnshaw Etzel by Father Moynihan, President of St. Thomas College, a discussion of college entrance requirements for mathematics opened by W. D. Reeve, College of Education, University of Minnesota, popular talks on mathematics by Jessie G. Quigley, College of St. Teresa, and five papers upon mathematical subjects by R. M. Mathews, Duluth Central High School, W. H. Kirchner of the college of engineering, University of Minnesota, R. A. Johnson, Hamline University, D. C. Kazarinoff, Carleton College, and W. H. Bussey, University of Minnesota.

The program was unusually well selected and presented and all papers proved of interest to the audience. Mr. Reeve, in opening the discussion of college entrance requirements in mathematics, urged the use of mental tests in addition to the method of certification and presented a considerable body of data which has grown out of his experience in this work.

Mr. Mathews's paper on "The graphics of intersections of line and conic" appears elsewhere in this issue of the MONTHLY.

Mr. Kirchner followed his discussion of last year upon the desirability of introducing Descriptive Geometry as a course in college mathematics, with a short paper upon "The co-point of three planes" in which he set up the conditions for the common point of three planes from the point of view of descriptive geometry and interpreted these in terms of the corresponding conditions from the view points of analytic geometry and algebra.

Dr. Johnson's paper dealt with the well-known theorem usually ascribed to

Fontené: The necessary and sufficient condition that the pedal circle of a point be tangent to the nine-point circle in that the point and its isogonal conjugate be collinear with the circumcenter. An alternative method of finding, with ruler and compasses, the intersections of a line with a conic given by five points was incidentally developed.

Mr. Kazarinoff's paper on "Some properties of focal conics of quadrics" appears elsewhere in this issue of the MONTHLY as "Dupin's Theorem."

President Moynihan of the College of St. Thomas related the events of the life of Father William Earnshaw Etzel in a very interesting manner, giving an insight into the motives and accomplishments of Father Etzel at once keen and sympathetic. Father Etzel was a charter member of the Section, an officer and an ardent helpful worker whose presence is greatly missed.

Mr. Swann's paper on "The diurnal variation of the atmospheric potential-gradient" is in abstract as follows:

Various phenomena in cosmical physics lend support to the view that the upper atmosphere is highly conducting as compared with the air near the earth. Thus, for example, considerations based on the ionization of the upper atmosphere by the sun's ultra-violet light show that this agency is capable of accounting for such a high conductivity that the resistance of a column of the upper air equal in length to the earth's quadrant would be no more than that of a column 3 cms. long and of the same cross section at the earth's surface. Thus, as far as this agency is concerned, the upper atmosphere may, for electrostatic purposes, be looked upon as a perfect conductor. Although the ultra violet light affects only the sunlit portions of the atmosphere, we have reason to believe that the corpuscular radiation responsible for the aurora is more evenly distributed.

If two concentric spheres be maintained at a difference of potential, the field at the surface of the inner sphere will, from symmetry, be the same at all points. If a dent be made in the outer sphere, the distance between the spheres at this point will be decreased, and since each sphere is at the same potential at all points, the *field* at the portion of the surface of the inner sphere under the dent will be stronger than the field elsewhere. This crude illustration suggests the view that if the upper atmosphere is in a highly conducting state, but the high conductivity on the sunlit side extends to lower levels than on the opposite side, the potential gradient should be higher in the former case than in the latter.

Here then would be an influence playing a part in the determination of the diurnal variation of the potential gradient, and such as to predict a maximum by day and a minimum by night, which condition is found, in a general way, to prevail. In its simplest aspects it would predict a diurnal variation of the air-earth conduction current-density of the same type as the potential-gradient. In practice it is found that the potential-gradient and conductivity at the earth's surface vary in opposite ways in such a manner as to secure greater constancy in the conduction current-density than in either of its constituent factors. However, it is by no means universally true that the conduction current-density remains even approximately constant; it is thus of interest to trace out in some-

what greater detail the consequences of the view outlined above, by working out an example where the change of conductivity with altitude is not absolutely sudden, as in the crude illustration cited, but takes place in a continuous manner.

In the problem considered, a distribution of potential-gradient over the surface of the earth is expressed in a series of Legendre coefficients in the form:

$$\left(\frac{\partial V}{\partial r}\right)_{r=a} = A_0 + A_1 P_1(\mu) + \cdots + A_n P_n(\mu) + \cdots \tag{1}$$

r, θ, being the polar coördinates of a point, and θ being measured from the line joining the center of the earth to that of the sun.

It is shown that a solution of the equation of continuity, subject to $V = 0$ at the sphere of radius a (the radius of the earth), and giving the distribution (1) over this sphere is given by:

$V =$ any function of ψ,

where,

$$\psi = -\frac{\sigma_0 A_0 a^2}{r} + \frac{\sigma_0 A_1}{3}\left(r - \frac{a^3}{r^2}\right) P_1(\mu) + \cdots + \frac{\sigma_0 A_n}{2n+1}\left[\frac{r^n}{a^{n-1}} - \frac{a^{n+2}}{r^{n+1}}\right] P_n(\mu) + \cdots,$$

provided that σ the conductivity is given by:

$$\sigma = \left(\frac{dV}{d\psi}\right)^{-1}, \text{ while}$$

σ_0 is the conductivity at the surface of the earth. As a particular example, the case

$$\left(\frac{\partial V}{\partial r}\right)_{r=a} = X_0 + X_0\alpha(1 + \cos\theta) + \cdots \tag{2}$$

is considered, α being a constant which, for illustration, is taken as 0.25. This expression corresponds to a decrease in $(\partial V/\partial r)_{r=a}$ from $\theta = 0$ to $\theta = \pi$. Values of σ and $\partial V/\partial r$ given respectively by:

$$\sigma = \sigma_0 e^{\beta\left[1 - \left\{\frac{a}{r} - \frac{a}{3(1+a)}\left(\frac{r}{a} - \frac{a^2}{r^2}\right)\cos\theta\right\}\right]} \tag{3}$$

and

$$\frac{\partial V}{\partial r} = X_0\left[(1+\alpha)\frac{a^2}{r^2} + \frac{\alpha}{3}\left(1 + \frac{2a^3}{r^3}\right)\cos\theta\right] e^{-\beta\left[1 - \left\{\frac{a}{r} - \frac{a}{3(1+a)}\left(\frac{r}{a} - \frac{a^2}{r^2}\right)\cos\theta\right\}\right]} \tag{4}$$

satisfy the necessary conditions for this case, β being constant. It is shown that if β is chosen so as to make σ increase by a factor of 30 at an altitude of 10 kilometers, the conductivity at such an altitude as is given by $r = 2a$, for example, is practically infinite for all values of θ; and, the distribution of $\partial V/\partial r$ in the region $r < 2a$ is sensibly the same as would have resulted from any other expression for σ which was sensibly the same as that chosen in the region $r < 2a$, even though, as r increased in value, this other value of σ gradually departed from the form (3) in such a way as to avoid the peculiarities associated with that form for large values of r.

It will be observed that the form (3) is such that σ increases more rapidly with r when $\theta = 0$ than for any other value of θ, and it is thus of the type to represent qualitatively the influence of the sun's ultra-violet light. The constant α may be taken as representing the intensity of the sun's effect. If there were no ultra-violet light, α would be zero, and (3) and (4) would indicate equal rates of increase of σ and of decrease of $\partial V/\partial r$ with altitude for all values of θ. Moreover, the potential-gradient would be constant over the earth's surface, as shown by (2).

The views outlined in the paper are applicable to the case of the solar eclipse, and are in line with experiment in predicting a diminution of the potential-gradient as a result of removal of the sun's rays.

Mr. Bussey's paper on "Methods of finding the square root of a number by means of a computing machine" gave two methods. (1) If a is an approximate value of the square root of a number $N = (a + b)^2$, the number $\frac{1}{2}[(N/a) + a]$ is a better approximation. This follows from the fact that

$$\tfrac{1}{2}[(N/a) + a] = (a + b) + b^2/2a$$

which is nearly equal to $(a + b)$ if b is small in comparison with a. The divisions involved in this method can be performed with ease on a computing machine. The method is equivalent to Newton's method of approximation. (2) The square root of a number less than 10,000 can be found by repeated subtraction by means of the following theorems:

(i) a^2 = the sum of a consecutive odd numbers beginning with unity.

(ii) $2abt + b^2$ = the sum of b consecutive odd numbers beginning with $2\,at + 1$.

Example:

529–100 = 429 } Two subtractions; therefore the first figure of
429–300 = 129 } the square root is 2.
129– 41 = 88 } Three subtractions; therefore the second figure of
88– 43 = 45 } the square root is 3. Answer, 23.
45– 45 = 0 }

The method can be extended so as to apply to numbers greater than 10,000 and to numbers which are decimal fractions.

Mr. Beal's paper reviewed *An introductory treatise on dynamical astronomy*, by H. C. Plummer (see this MONTHLY, June, 1919, pages 253–254).

Miss Quigley gave numerous illustrations of success she has attained by popular lectures in interesting students in mathematics and its study.

The program was followed by an excellent dinner at which the Section was guest to Hamline University. For the ensuing year R. A. JOHNSON was elected chairman, R. M. BARTON, secretary-treasurer, C. H. GINGRICH, C. H. YEATON, R. M. MATHEWS, executive committee.

It was voted that resolutions upon the death of Father Etzel be drawn up and placed in the minutes of the Section and that a copy of the same be sent to St. Thomas College.

R. M. BARTON, *Secretary-Treasurer.*

THE WORK OF THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

Since the publication of the statement concerning the work of the National Committe on Mathematical Requirements in the September 1919 number of the MONTHLY, the following events have occurred.

The office of the Chairman of the Committee has been located in the Musgrove Building, Hanover, New Hampshire. Mr. J. A. Foberg's office is at 3829 North Tripp Avenue, Chicago, Illinois.

The following members have been added to the Committee:

Mr. A. C. OLNEY, Commissioner of Secondary Schools, Sacramento, California.

Mr. P. H. UNDERWOOD, Ball High School, Galveston, Texas.

Miss EULA A. WEEKS, Cleveland High School, St. Louis, Missouri.

Mr. WALTER DOWNEY, of the English High School, Boston, Massachusetts, has been appointed by the New England Association of Teachers of Mathematics to take the place of Mr. G. W. EVANS, who is spending a year's leave in China.

A report on "The Reorganization of the Introductory Course in Secondary School Mathematics" by G. W. Evans has been received, and is being revised with a view to making it the basis of discussion on the part of teachers organizations throughout the country. A report by A. R. Crathorne on "Change of Mind between High School and College as to Life Work" has also been received; it is expected that this report will be published soon. Professor Crathorne's work on "A Critical Study of the Correlation Method as Applied to Grades" is progressing rapidly. An extended report on "Experimental Schools and Courses" by Mr. Raleigh Schorling and one by Mr. J. A. Foberg on "Mathematics for the Junior High School" are in preparation, as is also a preliminary report on the "Revision of College Entrance Requirements" by the Chairman. A comprehensive bibliography of the recent literature on the subject of the teaching of mathematics is being made under the direction of Professor David Eugene Smith.

The following further items regarding the work of the Committee may be of interest.

A letter addressed to some 30 publishing houses asking their coöperation in the work of the Committee has received a very generous response. As a result, the Committee now has a collection of almost 200 modern textbooks relating to secondary and elementary college mathematics.

The efforts of the Committee to make contact with organizations of teachers of mathematics throughout the country is also beginning to yield results. The directory of such organizations now includes about 40. Requests for speakers on the part of such organizations at meetings during October and November have been received from the Indiana State Teachers Association, Northwestern Illinois Teachers Association, New Jersey Association of Mathematics Teachers,

Iowa Association of Mathematics Teachers, New York State Teachers Association, Texas State Teachers Association, the High School Conference of the University of Illinois, the Association of Teachers of Mathematics in the Middle States and Maryland and the North Dakota Educational Association.

An effort is being made to get the work of the Committee before the public through the general educational periodicals of the country. Letters have been sent out to over 50 such periodicals. Those which have been heard from have given a very cordial response.

The Committee plans further to promote the formation of new organizations of teachers, where such organizations are needed and do not exist at the present time. These organizations may be sectional, covering a considerable area, or they may consist merely of local clubs which can meet at frequent intervals for the discussion and study of the problems of the Committee. It is hoped that such clubs can be organized in all the larger cities where they do not already exist.

It plans also to establish contact directly with individual teachers. The committee feels that this is necessary in addition to their work through organizations in order to induce such individuals to become active and in order to make the work through organizations effective. Plans for establishing this contact with individuals on a large scale are under consideration.

Organizations can be of assistance by sending to the Committee a statement of the name of the organization, its officers for the coming year, the time and place of its meetings and information regarding proposed programs. If any organization has within the last ten years issued any reports on topics connected with the work of the Committee, copies of such reports should, if available, be sent both to Mr. Young and Mr. Foberg. If this is impossible, a statement regarding the character and place of publication of any such reports would be welcome.

Individuals can be of assistance

1. By keeping the Committee informed of matters of interest that come to their notice;

2. By suggesting ways in which the Committee can be helpful;

3. By sending to the committee in duplicate reprints of any articles they publish on subjects connected with the Committee's work;

4. By furthering the work of the Committee among their colleagues, organizing discussions, etc.

It is not too much to say that the existence of this Committee with its present resources gives the teachers of mathematics, both individually and through their organizations, a unique opportunity to do really constructive work of the highest importance in the direction of reform. They can surely be counted on to make the most of this opportunity.

[*For later information see page 462.*]

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

In the first discussion this month, Mr. Kazarinoff gives simple demonstrations of several theorems on conic sections in space, suggested by the properties of focal conics of a system of confocal quadrics. Without access to original sources or other adequate references, the author was led to these theorems by direct study of Figs. 1 and 2 of the article. Though Mr. Kazarinoff's theorems lack entire novelty it is hoped that the directness and simplicity of his demonstrations may aid in making them more widely known and admired; and that students of conics may thereby be incited to look into the generalizations of Chasles and Plücker.

The second discussion is of interest as affording an instance of the need of a theorem in mathematics for the purpose of answering a particular question in an applied science. In connection with a condition for stability of thermodynamic equilibrium of a fluid phase of a two-component body Professor Trevor is led to a property of homogeneous functions. The result is itself of interest, and suggests the possibility of generalizations. Probably the denominator $x_3 + x_4$ might be altered to any linear function of the variables with like results. Do similar theorems exist for more general types of transformation? What are the corresponding facts for other isobaric functions than homogeneous? Can the facts in the present and other cases be deduced in any simpler way?

Direct interpretations of imaginary intersections of geometric loci, by operations on the figure itself, have frequently aroused interest. Several such treatments exist for determining complex roots of quadratic and cubic equations. Mr. Mathews gives in the third discussion a unified treatment, by which a number of such questions can be resolved by the application of essentially a single idea—his *virtual image* of a circle with regard to a straight line.

I. DUPIN'S THEOREM.[1]

By D. C. KAZARINOFF, University of Michigan.

It is well known that the consideration of the family of confocal quadrics

$$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}+\frac{z^2}{c^2+\lambda}=1 \quad (a^2>b^2>c^2), \qquad (F)$$

leads to the three focal conics:[2]

[1] Read before the Minnesota Section of the Mathematical Association of America, May 31, 1919.

[2] C. Smith, *An elementary treatise on Solid Geometry*, 10th ed., London, 1905, pp. 144, 145.

Ellipse: $$\frac{x^2}{a^2-c^2}+\frac{y^2}{b^2-c^2}=1, \qquad z=0, \tag{E}$$

Hyperbola:

$$\frac{x^2}{a^2-b^2}+\frac{z^2}{c^2-b^2}=1, \qquad y=0, \tag{H}$$

Imaginary conic:

$$\frac{y^2}{b^2-a^2}+\frac{z^2}{c^2-a^2}=1, \qquad x=0. \tag{I}$$

Furthermore the following property is well known:[1]

The locus of the vertices of the right circular cones which envelop the surfaces (F) *consists of the focal conics* (E), (H), *and* (I).[2]

Since the part of the plane $z=0$ interior to (E) and the part of the plane $y=0$ which does not contain the center and is bounded by (H) may both be regarded as limiting cases of surfaces (F),[3] the following particular property may be derived:

Property (A) (see Fig. 1): *Given any ellipse* (E) *and any hyperbola* (H) *in perpendicular planes, so placed that the foci of* (H) *are the vertices of* (E), *and vice versa. Then the locus of the vertices of right circular cones on the elliptic base* (E) *is the hyperbola* (H), *and vice versa.*[4]

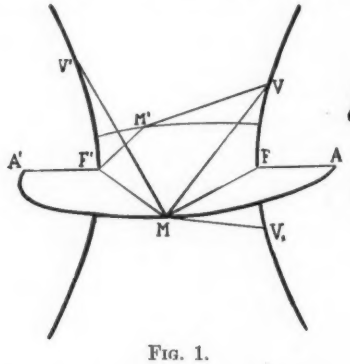

FIG. 1.

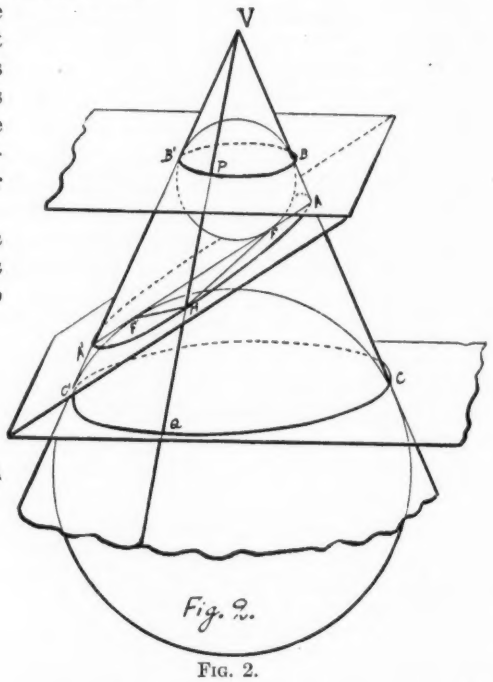

FIG. 2.

[1] C. Smith, *idem*, pp. 153–155. See also G. Salmon, *A treatise on conic sections*, 3d ed., London, 1855, p. 303.

[2] This result is due to Steiner, *Journal für die reine und angewandte Mathematik*, vol. 1 (1826), pp. 47 ff.—EDITOR-IN-CHIEF.

[3] C. Smith, *idem*, pp. 144, 145.

[4] This result is due to C. Dupin, *Correspondance sur l'école polytechnique*, vol. 2 (Jan., 1813), p. 424; in the *Correspondance* for April, 1804, Hachette attributed the result to Dupin.—EDITOR-IN-CHIEF.

My purpose is to present here an elementary proof of the property (A) and to deduce from it some other reciprocal properties of (E) and H.

Proof of property (A) (see Fig. 2):

Let V be the vertex of a right circular cone on the elliptic base $AA'M$ of which the vertices and the foci are respectively A, A' and F, F'. The points of contact of two spheres inscribed in the cone and touching the plane $AA'M$ will be the foci F, F'.[1]

The plane VAA' passes through the centers of the spheres and is therefore perpendicular to the plane $AA'M$.

Furthermore,

$$VA' - VA = B'A' - BA = A'F - AF = FF'.$$

Hence V is a point on an hyperbola with vertices and foci respectively at F, F' and A, A' in a plane perpendicular to the plane $AA'M$.[2] Since it can be proved in a similar way that from every point of the above hyperbola the ellipse $AA'M$ will be projected by a right circular cone, we have (Fig. 1):

The locus of the point V is an hyperbola with the same relative position to the ellipse $AA'M$ as (H) has with respect to (E).

Mutatis mutandis we can prove the second part of the property (A).

From Fig. 2 we have also:

$$MV + MF' = MV + MQ = VQ = \text{constant},$$

(b)

(c) $$MV - MF = MV - MP = VP = \text{constant}.$$

Property (B) (See Fig. 1):

V, V' being any two fixed points on different branches of the hyperbola (H), and M any point on the ellipse (E),

$$MV + MV' = \text{constant}.$$

Proof:

$$MV + MF' = \text{constant},$$

$$MV' + MF = \text{constant},$$

$$MF' + MF = AA' = \text{constant}.$$

Hence

$$MV + MV' = \text{constant}.$$

Property (C):

V, V_1 being any two fixed points on the same branch of (H), and M any point on the ellipse (E),

$$MV - MV_1 = \text{constant}.$$

Proof:

[1] G. Salmon, *idem*, footnote on p. 305.

[2] This property is due to G. P. Dandelin, *Nouv. Mém. Acad. Bruxelles*, volume 2 (1822), p. 172. —EDITOR-IN-CHIEF.

$$MV - MF = \text{constant.}$$

$$MV_1 - MF = \text{constant.}$$

Hence

$$MV - MV_1 = \text{constant.} \qquad \text{Q. E. D.}$$

Property (*D*):

M, M′ being any two fixed points on the ellipse (*E*), *and V any point on the hyperbola* (*H*),[1]

$$VM - VM' = \text{constant.}$$

Proof:

$$MV + MF' = M'V + M'F' = \text{constant.}$$

Hence

$$VM - VM' = \text{constant.} \qquad \text{Q. E. D.}$$

It can be proved without difficulty that the locus of the points having the property (*B*) is the hyperbola (*H*), as well as that the locus of the points having the property (*C*) is the ellipse (*E*).

II. A Property of Homogeneous Functions.

By J. E. Trevor, Cornell University.

When a body constituted of two independent component substances and subject to no mechanical or thermal separation of its parts is in a state of thermodynamic equilibrium, the body may exhibit distinct liquid or aëriform parts. Any such part has variable mass, composition, and thermodynamic state, and is termed a fluid "phase" of the body. When x_1, x_2, x_3, x_4 denote the volume, the entropy, and the component-masses of a two-component phase, the energy of the phase is a homogeneous function $E(x_1, x_2, x_3, x_4)$ such that

$$t \cdot E(x_1, x_2, x_3, x_4) = E(tx_1, tx_2, tx_3, tx_4), \tag{1}$$

where t is any positive number.[2] The specific volume, specific entropy, and specific component-masses of the body are defined by the equations

$$y_1 = \frac{x_1}{x_3 + x_4}, \qquad y_2 = \frac{x_2}{x_3 + x_4}, \qquad y_3 = \frac{x_3}{x_3 + x_4}, \qquad y_4 = \frac{x_4}{x_3 + x_4} = 1 - y_3.$$

When $t = 1/(x_3 + x_4)$, and X is written for $x_3 + x_4$, the equation (1) becomes

$$E = X \cdot E(y_1, y_2, y_3, 1 - y_3)$$

$$= X \cdot e(y_1, y_2, y_3), \tag{2}$$

where the "specific energy" e of the phase is a function of the three variables y_1, y_2, y_3.

[1] Properties (*B*), (*C*) and (*D*) are due to Dupin, *Correspondance sur l'école polytechnique*, Jan., 1807, p. 218, and Jan., 1813, p. 424. For further references in this connection, and to generalizations by Chasles and Plücker, the *Encyclopédie des Sciences Mathématiques*, tome III, vol. 4, pp. 81–86, and tome III, vol. 3, pp. 51–52, may be consulted.—Editor-in-Chief.

[2] Functions which possess this restricted form of homogeneity are termed "positively homogeneous" by Bolza, *Lectures on the Calculus of Variations*, Chicago, 1904, p. 119.—Editor.

If we write Taylor's expansion in the notation $\delta E = \delta^1 E + \delta^2 E + \cdots$, where $\delta^n E$ is the sum of terms of the nth order, we have that the criterion of the stability of the thermodynamic equilibrium of a fluid phase is $\delta^2 E \geq 0$, where the sign of equality holds only when the variations of the variables satisfy the conditions

$$\frac{\delta x_1}{x_1} = \frac{\delta x_2}{x_2} = \frac{\delta x_3}{x_3} = \frac{\delta x_4}{x_4}. \tag{3}$$

In the desire to express the conditions of stability in terms of derivatives of the specific energy $e(y_1, y_2, y_3)$, it becomes necessary to establish a relation between the sum of terms $\delta^2 E$ and the variations of the function e. Giving the four variables x_i independent increments δx_i, the functions E, e in (2) obtain increments δE, δe, and we have

$$\delta E = e\delta X + X\delta e + \delta X \delta e,$$

or

$$\delta^1 E + \delta^2 E + \cdots = e\delta X + X\cdot\delta^1 e + X\cdot\delta^2 e + \cdots + \delta X\cdot\delta^1 e + \delta X\cdot\delta^2 e + \cdots. \tag{4}$$

The first member of this equation is a power series in δx_1, δx_2, δx_3, δx_4, with constant coefficients. The second member is a power series in δy_1, δy_2, δy_3, δX, with constant coefficients. Now, since $x_i = X y_i$, the variables δx_i have the values

$$\delta x_i = (X + \delta X)\delta y_i + y_i \delta X \qquad (i = 1, 2, 3, 4). \tag{5}$$

where $y_4 = 1 - y_3$ and $\delta y_4 = -\delta y_3$. If we replace the δx_i in the first member of (4) by these values, both members of (4) will be power series in δy_1, δy_2, δy_3, δX.

The first term of the first member of (4) is a sum of terms $\delta^1 E = \Sigma A_i \delta x_i$, or by (5),

$$\delta^1 E = X\cdot\Sigma A_i \delta y_i + \delta X\cdot\Sigma A_i \delta y_i + \delta X\cdot\Sigma A_i y_i. \tag{6}$$

The first, and no other, of these three expressions contains δy_i with no factor δX. No terms in δy_i alone occur in $\delta^2 E$, $\delta^3 E$, $\cdots$, since by (5) the terms occurring there are at least quadratic in the variables. So the first expression in the right-hand member of (6) is equal to the sum of the terms in δy_i in the second member of (4),

$$X\cdot\Sigma A_i \delta y_i = X\cdot\delta^1 e. \tag{7a}$$

It follows that $\delta^1 e = \Sigma A_i \cdot \delta y_i$; wherefore the second expression in (6) is

$$\delta X\cdot\Sigma A_i \delta y_i = \delta X\cdot\delta^1 e. \tag{7b}$$

The third expression in (6) is in δX. No terms in δX alone occur in $\delta^2 E$, $\delta^3 E$, $\cdots$, since these terms are at least quadratic. So the third expression in (6) is equal to the term in δX in the second member of (4),

$$\delta X\cdot\Sigma A_i y_i = e\cdot\delta X. \tag{7c}$$

On substituting these three results, (6) becomes

$$\delta^1 E = (X + \delta X)\delta^1 e + e\delta X. \tag{8}$$

On subtracting (8) from (4), member by member, there remains

$$(9) \qquad \delta^2E + \delta^3E + \cdots = X\cdot\delta^2e + X\cdot\delta^3e + \cdots + \delta X\cdot\delta^2e + \delta X\cdot\delta^3e + \cdots.$$

The first term of the first member of (9) is a sum of terms $\delta^2E = \Sigma A_{ij}\delta x_i\delta x_j$, or by (5)

$$\delta^2E = \Sigma A_{ij}[(X + \delta X)\delta y_i + y_i\delta X][(X + \delta X)\delta y_j + y_j\delta X].$$

Expanding the indicated product,

$$(10) \quad \delta^2E = X^2\cdot\Sigma A_{ij}\delta y_i\delta y_j + 2X\delta X\cdot\Sigma A_{ij}\delta y_i\delta y_j + (\delta X)^2\cdot\Sigma A_{ij}\delta y_i\delta y_j + X\delta X\cdot\Sigma A_{ij}(y_i\delta y_j + y_j\delta y_i) + (\delta X)^2\cdot\Sigma A_{ij}(y_i\delta y_j + y_j\delta y_i) + (\delta X)^2\cdot\Sigma A_{ij}y_iy_j.$$

The first, and no other, of these six expressions is in $\delta y_i\delta y_j$. No terms in this product alone occur in δ^3E, δ^4E, $\cdots$, since by (5) the terms occurring there are at least cubic in the variables. So the first expression in (10) is equal to the sum of terms in $\delta y_i\delta y_j$ in the second member of (9),

$$(11a) \qquad X^2\cdot\Sigma A_{ij}\delta y_i\delta y_j = X\cdot\delta^2e.$$

It follows that $\delta^2e = X\cdot\Sigma A_{ij}\delta y_i\delta y_j$; wherefore the second and third expressions in (10) are

$$(11b) \qquad 2X\delta X\cdot\Sigma A_{ij}\delta y_i\delta y_j = 2\delta X\cdot\delta^2e,$$

$$(11c) \qquad (\delta X)^2\cdot\Sigma A_{ij}\delta y_i\delta y_j = \frac{(\delta X)^2}{X}\delta^2e.$$

The fourth expression in (10) is in $\delta X\delta y_j$ (respectively $\delta X\delta y_i$). No terms in this product occur in δ^3E, δ^4E, $\cdots$, since these terms are at least cubic. So the fourth expression is equal to the terms in this product in the second member of (9). But such terms are absent; thus

$$(11d) \qquad X\delta X\cdot\Sigma A_{ij}(y_i\delta y_j + y_j\delta y_i) = 0.$$

It follows that $\Sigma A_{ij}(y_i\delta y_j + y_j\delta y_i) = 0$, and hence that the fifth expression vanishes,

$$(11e) \qquad (\delta X)^2\cdot\Sigma A_{ij}(y_i\delta y_j + y_j\delta y_i) = 0.$$

The sixth expression in (10) is in $(\delta X)^2$. No terms in this square occur in δ^3E, δ^4E, $\cdots$, since these terms are at least cubic. So the sixth expression is equal to the terms in $(\delta X)^2$ in the second member of (9). But such terms are absent, so that

$$(11f) \qquad (\delta X)^2\cdot\Sigma A_{ij}y_iy_j = 0.$$

On substituting these six results, (10) becomes

$$\delta^2E = [X + 2\delta X + (\delta X)^2/X]\delta^2e.$$

I.e., the expressions $\delta^2 e$ and $\delta^2 E$ are connected by the relation

$$\delta^2 E = \frac{(X + \delta X)^2}{X} \delta^2 e. \tag{12}$$

Since the coefficient of $\delta^2 e$ in (12) is positive, it follows that the criterion of stability $\delta^2 E(x_1, x_2, x_3, x_4) \geq 0$ and the criterion $\delta^2 e(y_1, y_2, y_3) \geq 0$ are equivalent. In both forms of the criterion the sign of equality holds when the variations of the variables satisfy the conditions (3). When these conditions are satisfied, e is constant and the equation

$$\delta E = (X + \delta X)\delta^1 e + e\delta X + \frac{(X+\delta X)^2}{X} \delta^2 e + \cdots$$

reduces to $\delta E = e\delta X$, which represents an obvious physical fact. Since the foregoing method of obtaining the results (8) and (12) is tedious, I should be glad to learn of a simpler way of finding them.

III. Graphical Constructions for Imaginary Intersections of Line and Conic.[1]

By R. M. Mathews, University of Minnesota.

Introduction.—Several methods commonly known for the graphical solution of a quadratic equation are incomplete, as they are explained only for real roots. The object of this paper is to complete these constructions and to generalize them to find graphically the intersections of an arbitrary line with any conic.

Intersection of a circle and a line.—When a line l cuts a circle O the length of the chord formed is $2\sqrt{r^2 - d^2}$, where r is the radius of the circle and d is the distance from the center O to the chord.

When the line does not cut the circle in real points, $d > r$, the sign of $r^2 - d^2$ is negative and the length of the chord is imaginary. Let us replace the given circle by a second which cuts the line in real points such that the length of the chord formed is $2\sqrt{d^2 - r^2}$. Draw OM perpendicular to the line to cut the circle at T and l at M, and extend OM to O' so that $MO' = r$ (Fig. 1); with O' as center draw a circle tangent to the given one at T. The length of the chord which it cuts on l is $2\sqrt{d^2 - r^2} = 2i\sqrt{r^2 - d^2}$.

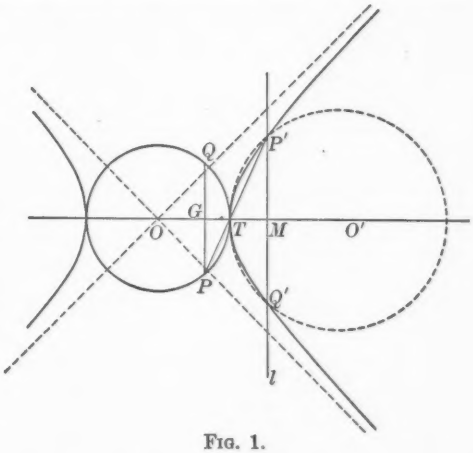

Fig. 1.

[1] Read before the Minnesota Section of the Mathematical Association of America, May 31, 1919.

If the points of intersection be P' and Q', they may represent the imaginary points in which the line cuts the given circle, and $P'Q'$ the length of the imaginary chord.

Let us call this second circle O' the *virtual image* of the given circle with regard to the line.

Applications in analysis.—(i) The graph of

$$x^2 + y^2 + 2gx + 2fy + k = 0 \tag{1}$$

is a circle with center at $(-g, -f)$ and radius $+\sqrt{g^2 + f^2 - k}$. The graph of

$$\alpha x + \beta y + \gamma = 0 \tag{2}$$

is a straight line with intercepts $(-\gamma/\alpha, 0)$, $(0, -\gamma/\beta)$. The algebraic solution of the system (1), (2) leads to a quadratic equation in x

$$ax^2 + bx + c = 0. \tag{3}$$

whence

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}. \tag{4}$$

The values of y are found by substitution of these x's in (2). Let P and Q be the points of intersection and M the midpoint of the chord PQ.

When the points P and Q are real we ordinarily read out their coördinates directly and so neglect to observe that in (4) the number $-b/2a$ is the abscissa of M while $\sqrt{b^2 - 4ac}/2a$ is half the difference of the abscissæ of P and Q. This guides us in reading out the solutions for the case of imaginary points of intersection. Let the virtual image of (1) cut (2) in points P' and Q', with M' the midpoint of chord. For the intersection of this image with (2), we have

$$x' = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a},$$

and accordingly the values for x are

abscissa of $M' \pm i/2 \times$ (difference of abscissæ of P' and Q'),

while the values for y are

ordinate of $M' \pm i/2 \times$ (difference of ordinates of P' and Q').

(ii) The algebraic solution of the system of equations

$$x^2 + y^2 + 2g_1x + 2f_1y + k_1 = 0, \tag{5}$$

$$x^2 + y^2 + 2g_2x + 2f_2y + k_2 = 0, \tag{6}$$

is effected by subtracting the one from the other and solving the resulting linear equation with (1). The graph of this linear equation is the radical axis of the two circles given by the equations. Thus the graphical solution consists in constructing the radical axis of the circles and finding its intersections with one of them.

(iii) A combination of the preceding results enables us to construct a length to represent the imaginary tangent from a point to a circle and to find a point whence we can read the complex coördinates of the imaginary points of tangency.

Graphical solution of a quadratic equation.—If x_1 and x_2 are the roots of the quadratic equation

$$x^2 + px + q = 0,$$

then

$$x_1 + x_2 = -p, \qquad x_1 \cdot x_2 = q.$$

Take OM on the x-axis equal to $-p/2$; on the y-axis mark two points H and K such that $OH \cdot OK = q$. Draw that circle through H, K which has its center on the line through M parallel to OY. If this circle cuts OX in x_1 and x_2, then $x_1 \cdot x_2 = OH \cdot OK = q$, and $\frac{1}{2}(x_1 + x_2) = -\frac{1}{2}p$. Thus the abscissæ x_1 and x_2 are roots of the quadratic.

When the circle fails to cut the x-axis, construct its virtual image with regard to the axis. It will cut the line in two points x, and x_2 and the roots of the quadratic are

$$OM \pm \tfrac{1}{2}i\,|\,x_1 - x_2\,|\,.$$

The proxy of a circle.—To extend these developments to any conic we examine further the geometry of a circle and its virtual image. The point T is a center of similitude of the two circles (Fig. 1). Points on the image can be constructed by drawing any radius OX and then line XT to be cut in X' by a radius $O'T'$ parallel to OX. In particular, on OT take OG as a third proportional to OM and OT. Draw chord through G perpendicular to OT to cut circle at P; then PT cuts l at P' on l.

Secondly, each line of the pencil of parallels to l has a virtual image of the circle O and is cut by it in two points P' and Q'. The locus of P' is an equilateral hyperbola, with center at O, vertex at T, and with OT as transverse axis. Accordingly, each line parallel to l either cuts the circle in real points or cuts the hyperbola in points such that the discriminant for the solution of line and hyperbola is the negative of the discriminant for the solution of line and circle. For this reason it is proposed to call the hyperbola the *proxy* of the circle for the direction l.

Finally, the hyperbola can be plotted from the circle by taking for each line l a corresponding point G and proceeding as explained above.

Intersection of an ellipse and a line.—Consider a system of a circle, a line, the virtual image and the proxy of the circle with regard to the line. Rotate the system around a convenient diameter of the given circle, and project orthogonally upon the initial plane. The two circles become ellipses, the line a line, and the proxy equilateral hyperbola becomes a general hyperbola, because this transformation leaves the line at infinity invariant. For this reason parallel lines go into parallel lines, and similar triangles become similar triangles. Thus, all constructions that depend upon relations of similarity and proportion in the original figure are preserved, with the modification that orthogonal diameters become conjugate diameters.

The details for graphical solution of an ellipse with a line may now be recounted (Fig. 2). Across the curve draw two chords parallel to l and join their midpoints, obtaining the diameter conjugate to l. Let it cut the conic at T' and T, and cut l at M. Bisect $T'T$ for center O. Locate G on OT so that OG is a third proportional to OM and OT; through G draw a chord parallel to l and project its ends through T upon l as P' and Q'. Now the abscissæ of the points of intersection of the line and the ellipse are

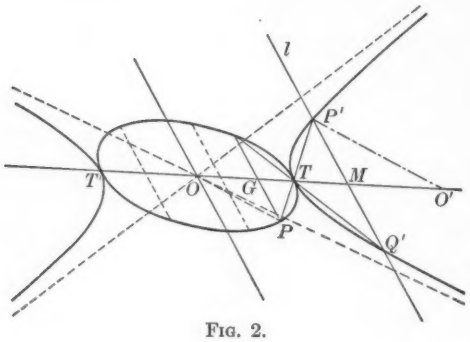

FIG. 2.

$$\text{abscissa of } M \pm \tfrac{1}{2}i \times (\text{difference of abscissæ of } P' \text{ and } Q'),$$

and the ordinates are

$$\text{ordinate of } M \pm \tfrac{1}{2}i \times (\text{difference of ordinates of } P' \text{ and } Q').$$

The center of the ellipse which is the virtual image of the given one is on OM extended to O' with $MO' = OT$. This image ellipse can be traced by drawing secants through T. If such an one, s, cuts the ellipse at R, draw $O'R'$ parallel to OR to cut s at R', a desired point. Or draw two conjugate diameters through O' to correspond to a convenient pair in the initial curve, and then plot the image with a trammel.

The locus of the points P' and Q' as line l moves parallel to itself is the hyperbola proxy for the ellipse. It can be plotted by locating P' and Q' for l in a series of positions; or we can remark that we have three points, T and one pair P', Q', and a pair of conjugate diameters, whence the curve can be constructed in a variety of ways.

Intersection of an hyperbola and a line.—We have just seen that for an ellipse and a pencil of parallels the proxy is an hyperbola. This hyperbola cuts a line of the pencil in two points such that in the analytic expression for their coördinates the discriminant is the *negative* of the discriminant for said line and the ellipse. Accordingly, those lines which the ellipse cuts in real points the hyperbola cuts in points with the corresponding complex coördinates. Thus the proxy of a hyperbola is that ellipse for which it is proxy, and all the constructions of the preceding paragraph can be applied to the case of a line and hyperbola.

Intersection of a parabola and a line.—The tangent to a parabola at a point T separates all lines parallel to it into two classes, those which cut the curve in real points and those which do not. Let P and Q be points on the curve with chord PQ parallel to the tangent at T (Fig. 3). Extend PT to P' with $P'T = PT$. Locate Q' similarly. Then $PQP'Q'$ is a parallelogram with PQ and $P'Q'$ equally distant from T. When this construction is repeated for every line PQ, parallel

to the tangent at T, the locus of P' and Q' is a parabola congruent and symmetrical to the first through T, and with the midpoints of the parallel chords collinear on a diameter of the curves. This new curve cuts in real points all lines parallel to tangent at T, that the first did not, and conversely.

To solve graphically for imaginary points of intersection of a line and parabola: draw two chords parallel to the line and join their midpoints obtaining a diameter to cut curve in T and l in M. On MT take $TO = MT$ and through O draw PQ parallel to l. Then PT cuts l in P' and QT cuts it in Q', and the coördinates of the imaginary points of intersection are read out as before.

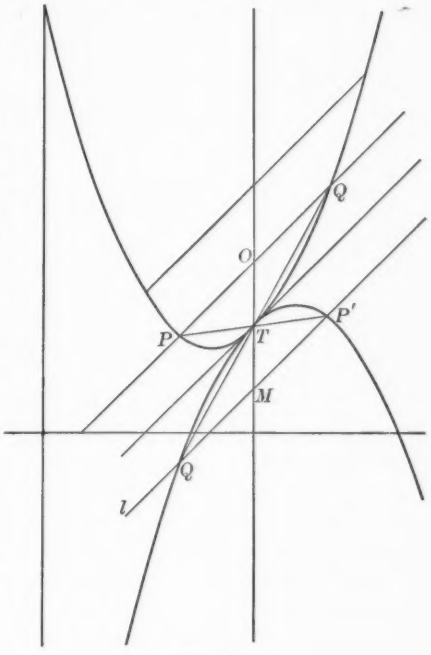

FIG. 3.

Solution of a quadratic equation.— Two commonly used graphical solutions of a quadratic equation

$$x^2 + px + q = 0 \tag{1}$$

depend on parabolas. In the first we plot the function

$$y = x^2 + px + q \tag{2}$$

and read out the intercepts on the x-axis. The failure of this method when the roots are complex can be remedied by drawing a line parallel to the x-axis with the vertex of the curve half way between them and proceeding as in the previous paragraph.

In the second method, we remark that the function (2) can be regarded as the sum of the expressions

$$y_1 = x^2 \qquad \text{and} \qquad y_2 = -px - q.$$

Thus the parabola $y_1 = x^2$ will be the same for all quadratics and is graphed once for all. Each of the two points in which the line $y_2 = -px - q$ cuts the curve has an abscissa x which makes $y_1 = y_2$ and so is a root of (1). With the addition of the methods of the preceding section we have a construction that solves every quadratic equation graphically.

Note: The topic of this paper is discussed in J. G. Hamilton and F. Kettle, *Graphs and Imaginaries*, London, Arnold, 1904; and A. Schultze, *Graphic Algebra*, New York, Macmillan, 1908.—EDITOR-IN-CHIEF.

RECENT PUBLICATIONS.

REVIEWS.

Vital Statistics, An Introduction to the Science of Demography. By G. C. WHIPPLE, Professor of Sanitary Engineering in Harvard University. New York, Wiley, 1919. xii + 517 pages. 12mo. Price $4.00.

This interesting and instructive book was not written primarily for mathematicians, but it stresses the need of mathematics and an intelligent, discriminating use of mathematics. The preface reads: "This book is written for students who are preparing themselves to be public-health officials and for public-health officials who are willing to be students . . . it emphasizes the need of using vital statistics with truth, with imagination and with power."

Printed in very attractive form, with numerous diagrams, tables, and illustrative matter (with 529 pages, but only $\frac{3}{4}$ inch in thickness), the book should fulfill its mission; it should also make an appeal to the general public; and, indeed, to mathematicians who are interested in the social sciences.

The mathematics involved runs from arithmetic up to the theory of probability and to correlation. As an application of Bernoulli's Theorem in probability, an epidemic of typhoid is postulated for a city where milk is supplied by two dealers—page 399. If the epidemic is due to the water supply of the city, each dealer should have among his customers about the same percentage of typhoid cases—other circumstances being the same. The problem is to determine how many cases above his proportionate number a milk dealer must have in order that suspicion may properly fall upon the milk he sells.

The author condenses in 60 pages some of the elements of probability, correlation, and life insurance—trying to impart some information to students not well trained in mathematics. In this condensation, certain infelicities have made their appearance. Frequency and probability are not adequately distinguished (p. 398). The so-called "probable error" is confused with the most probable error (pp. 390, 391). The name "curve of error" is applied to a cumulative curve of error (pp. 386, 387), and the latter is improperly described at the foot of p. 390. The "expectation of life" is confused with the "most probable lifetime" on p. 422, but on pp. 427, 428, the proper distinction is made. The Combined Experience Mortality Table, mentioned on p. 431, was based upon the data of British companies instead of American companies, and the date of the Northampton Table, given on p. 431 as 1762 is given by Henry Moir as 1783. It is not clear what distinction the author had in mind on p. 410 between Pearson's coefficient of correlation and Galton's. That the graphical method of getting the coefficient of correlation on pp. 412, 413, is defective, can be seen by supposing that each wheat-price were raised ten shillings—this would not alter the coefficient of correlation, but the computation on p. 413 would be altered. On p. 411,

the figure 0.54 is said to represent "low correlation." In a probability table on p. 392, the figure 0.5872 appears instead of 0.5817, and on p. 393 the figures 0.58 and 0.77 appear instead of 0.57 and 0.78, respectively. In the solution of the problem on p. 399, relating to the milk dealers and mentioned above, hardly enough allowance has been made for natural dispersion.

No attempt was made by the reviewer to check the many tables and statistical data in the book, but in glancing at the population table on p. 158, it was noticed that the population of Cleveland, Ohio had been used also for Cincinnati, Ohio—evidently a typographical error.

Vital Statistics is arranged with a set of exercises and questions at the end of each chapter, so that it may be used as a text-book. Such a stimulus to thought can not be too highly recommended, even for books not intended primarily as texts. In these questions, numerous references to the literature of the subject are given.

There is also a short bibliography in Appendix I. Here misprints appear in the names of Willford I. King and Harald Westergaard. Another book of Westergaard's, also published by Fischer, is *Die Grundzüge der Theorie der Statistik*, a book of some interest to mathematicians. E. Davenport's *Principles of Breeding*, with Appendix by H. L. Rietz (Ginn & Co.) may be added to the list; also, Merriman's *Text-Book on the Method of Least Squares* (Wiley). A more extended bibliography, for advanced work, would have included the names of Czuber, Poincaré, Borel, Bachelier, Bruns, Blaschke, Carvallo, Markoff, Montessus de Ballore, and many of the older authors. Among periodicals, *Biometrika* might well be mentioned. Appendices II and III give model state laws for reports of sickness, births and deaths, and standard certificates of birth and death. Appendix IV is a 5-place table of logarithms. Finally, there is an Index.

The following is a condensation of the "Contents" as listed on pp. vii–xii. For mathematicians, Chapters II, XII, XIII, XIV, will be of special interest,—as showing how mathematics is applied.

Contents. Chapter I. Demography: History of statistics—Vital Statistics; The statistical method. II. Statistical Arithmetic: Collection of data; Tabulation; Precision and accuracy; Logarithms; The slide-rule; Classification and generalization; Averages; The moving average; Mechanical devices. III. Statistical Graphics: Types of diagrams; Graphical deceptions; Summation diagrams; Polar coördinates; Double coördinate paper; Logarithmic cross-section paper. IV. Enumeration and Registration: United States census; Credibility of census returns; Births, deaths, marriages, sickness. V. Population: Estimation of population; Arithmetical increase—Geometrical increase; Decreasing rate of growth—Difference between estimate and fact; Population of U. S. cities (a table); Classification of population; Errors due to use of round numbers; Other sources of error; Standards of age distribution. VI. Death-rates, Birth-rates, Fecundity. Marriage-rates; Divorce-rates; Marriage-rates, birth-rates and death-rates in Sweden; Birth; rates, death-rates, marriage-rates and divorce-rates in Massachusetts. VII. Specific Death-Rates: Age, sex, marital condition, nationality; Chronological changes; Particular diseases. VIII; Causes of Death: Nosology; International list of causes of death; Present-day classification. IX. Analysis of Death-Rates: Two methods. X. Statistics of Particular Diseases: Tuberculosis; Diphtheria; Typhoid fever; Cancer. XI. Studies of Deaths by Age Periods: Infant mortality, its chronological reduction; Pre-natal deaths; Expectation of life at different ages; Causes of infant deaths; Maternal mortality; Diseases of early childhood; Proportionate mortality; Median age of persons living. XII. Probability: Natural frequency; Coin tossing; Chance; Binomial theorem; Chance and natural phenomena; Frequency curves, including skew curves;

Frequency curves shown by summation diagrams—Deviation from the mean; Standard deviation; Coefficient of variation; Computation of coefficient from grouped data; Probable error; Doubtful observations; The probability scale; Probability cross-section paper. Another use of probability; The frequency curve as a conception. XIII. Correlation: Correlation and causality; Laws of causation; Methods of correlation; Galton's coefficient of correlation; Example of low correlation—Correlation shown graphically; Correlation table; Use of mathematical formulæ; Secondary correlation; The lag—Coefficient of correlation and the lag; Other secondary correlations; The epidemiologist's use of correlation. XIV. Life Tables: Probability of living a year; Mortality tables; Most probable life-time; "Vie probable"; Expectation of life; Comparison of the three methods; Life-tables based on living populations; Mathematical formulæ; Early history of life-tables; Recent life tables; United States Life Tables: 1910; A few comparisons. XV. A Commencement Chapter: Military statistics; Industrial diseases; Accidents; Infantile paralysis; Sanitary index.

EDWARD L. DODD.

The Sumner Line, or Line of Position as an Aid to Navigation. By GEORGE C. COMSTOCK. New York, Wiley. 6 + 70 pages. 12 mo. Price $1.25.

This is one of a dozen or more of little books on navigation that have appeared within the past two years as a result of the increased activity of the Navy and the Merchant Marine. Many teachers of the subject have apparently felt that the standard American text book on navigation, "The American Practical Navigator," originally by Nathaniel Bowditch, is, in some respects, open to criticism, and they have tried to improve upon the explanations and methods used in various parts of the book.

Professor Comstock has confined his book to problems connected with the determination of the Line of Position, the method of locating a ship at sea, which has been adopted by the U. S. Navy.

Presuming familiarity on the part of the reader with the elements of navigation, the Nautical Almanac, and such instruments as the sextant, compass, chronometer, and log, the author begins with an explanation of the sub-solar point, *i. e.*, the point on the earth directly under the sun (or any other heavenly body). He then takes up the Circle of Equal Altitude, and the location, by the methods of St. Hilaire and de Aquino, of the Line of Position or that part of the Circle of Equal Altitude which is in the vicinity of the observer's ship.

The theory of the spherical traverse tables is fully explained, and their use is illustrated not only in finding the azimuth and the "calculated" altitude, but also in solving such problems as finding the amplitude, the time when a star will cross the Prime Vertical, and the identification of an unknown star.

One of the novel features of the book is the method of finding the coördinates of the intersection of two Lines of Position. This is commonly done at sea by plotting the lines on a chart. In Bowditch and in other epitomes, methods, based on the solution of right triangles, are given for computing the coördinates of intersection when a chart is not available. Professor Comstock has resorted to analytical geometry and has expressed his results in terms which can be handled by the use of plane traverse tables. This is of interest from a mathematical point of view, but probably will not become popular with practical navigators because of the many possibilities of mistakes, especially in algebraic signs.

Another feature of the book is the revival of horizon observations. Methods of determining the longitude from sunrise and sunset observations used to be given in the old epitomes, but, on account of the uncertainty of refraction near the horizon, those methods were omitted years ago. Professor Comstock has prepared a special table of Horizon Corrections for different temperatures and pressures, which, under normal conditions at sea, ought to enable the navigator to get fairly good results from horizon sights.

Throughout the book Forms are used for the systematic arrangement of the computations, and, to accompany the book, pads of Blank Forms have been prepared for working the St. Hilaire or de Aquino methods of finding the Line of Position. On each blank is a skeleton chart for plotting the lines.

F. SLOCUM.

NOTES.

In his *Varia Socratica*,[1] pages 151–155, A. E. Taylor discusses the familiarity of Socrates with mathematical science.

A new edition of the *Encyclopedia Americana* is nearing completion and is especially noteworthy here on account of the valuable mathematical articles contained therein. The edition is expected to comprise thirty volumes and the mathematical articles were prepared by a number of different American authors.

The following memoirs on ULISSE DINI have been recently published: By L. Bianchi in *Rendiconti della r. accademia dei Lincei, scienze fisiche*, Roma, 1919, (10 pages); by F. d'Arcais in *Atti del r. istituto Veneto di Scienze, lettere edarti*, Venezia, 1919 (7 pages).

A compiler of a mathematical dictionary might find the following recent volume in the Manuali Hoepli series of some service: D. M. Mele, *Dizionario internazionale di aeronavigazione e costruzioni aeronautiche, italiano, francese, inglese, tedesco con indice delle quattro lingue in alfabeto unico.* Milano, 1919. (7 + 227 pp.).

A new work by Professor A. N. Whitehead, entitled *An Enquiry concerning the Principles of Natural Knowledge*, has been published by the Cambridge University Press 12s. 6d. The four parts of the book deal with: The traditions of science, The data of science, The method of extensive abstraction, The theory of objects. In the investigation the scientific concepts of space and time are considered as the first outcome of the simplest generalization from experience.

The interesting and attractive first two numbers of *Norsk Matematisk Tidsskrift*, the official organ of the recently organized Norwegian Mathematical Society (*1919*, 324) are heartily welcomed by the MONTHLY. The first number, of 32 pages, contains articles on Sylow and his scientific work, on number theory polynomials and on an elementary proof of Kepler's first and second laws, besides brief notes, problems for solution, and news of the Society (which had 375 members on February 1, 1919). As inserts are also a fine portrait of Sylow and the laws of the Society. In addition to more problems, notes concerning the Society and its members, and book reviews, the second number, of 48 pages, contains articles on Indian mathematics, application of a formula for inversion to the theory of numbers, distribution of chance errors in observation, and graphical integration. The articles are written in Norwegian, Swedish, French, and German.

ARTICLES IN CURRENT PERIODICALS.

BOLLETTINO DI BIBLIOGRAFIA E STORIA DELLE SCIENZE MATEMATICHE, Volume 20, No. 3, 1918: "Sulla significato della parola 'prospettiva' usata nella legge sulla conservazione dei monumenti" by G. Pittarelli, 65–76; 17 reviews, 76–94 [including reviews by G. Loria of

[1] First series (St. Andrews University Publications, No. IX), Oxford, Parker, 1911.

Miller's *Historical Introduction to Mathematical Literature* (New York, 1916), of Cajori's *William Oughtred, a great seventeenth-century teacher of mathematics* (London and Chicago, 1916), and of *Fundamental Conceptions of Modern Mathematics* by Richardson and Landis (Chicago, 1916)]; "Notizie," 94–96.

COMMON GROUND, West Somerville, Mass., Volume 1, April, 1919: Mathematics in the elementary school and the junior high school," 7–15 [Report of the Committee on Curriculum of the Massachusetts Teachers' Federation].

INDUSTRIAL ARTS MANAGEMENT, New York, volume 8, February, 1919: "Mathematics for the apprentice," 58–62.

JOURNAL OF THE INDIAN MATHEMATICAL SOCIETY, volume 11, no. 3, June, 1919: "Some definite integrals" by S. Ramanujan, 81–87; "Fractional differentiation" by M. T. Naraniengar, 88–95; "On a system of three similar triangles, connected with a given triangle," 96–99; "The theory of linear systems of points," 100–108; Addition formulas for $sn(u_1 + u_2 + u_3)$ etc." by F. H. V. Gulasekharam, 109–111; "On question 963" by V. R. Aiyar, 111–112; "Astronomical notes" by T. P. Sastri, 113; Solutions and questions for solution, 114–120; Periodicals received, i–iii.

MATHEMATICAL GAZETTE, Volume 9, July, 1919: "Mathematics in business and industry" by H. T. H. Piaggio, 361–364; "Gleanings far and near," 364, 368, 371; "Extension of the theory of inversion to conics" by J. J. Milne, 365–368; "Notes for lessons introductory to differential geometry" by E. H. Neville, 369–371; [Greeks, and the theorem coaxial triangles are copolar] by J. J. Milne, 372–373; "Geometrical construction for the trisection of an angle to any required degree of accuracy" by D. F. Ferguson, 373; "Reviews," 374–376.

MESSENGER OF MATHEMATICS, Volume 48, no. 4, August, 1918: "On Nielsen's functional equations" by G. N. Watson, 49–53; "On a simple summation of the series $\sum_{s=0}^{n-1} e^{2s^2\pi i/n}$" by L. J. Mordell, 54–56; "Radiation from a moving magneton" by H. Bateman, 56–64—No. 5, September: "Radiation from a moving magneton," 65–76; "On a diophantine problem" by H. Holden, 77–80—No. 6, October: "On a diophantine problem" by H. Holden, 81–87; "A simple proof of the fundamental equality in the theory of the gamma function" by S. Pollard, 87–89; "Notes on some points in the integral calculus" by G. H. Hardy, 90–96—No. 7, November: "Notes on some points in the integral calculus" (50) by G. H. Hardy, 97–100; "Method of expressing the cross-ratio of the range given by the roots of a biquadratic equation in terms of an auxiliary angle connected with the roots of the reducing cubic of the biquadratic" by A. Lodge, 100–107; "Notes on some points in the integral calculus" (51) by G. H. Hardy, 107–112—No. 8, December, 1918: "General pentaspherical coördinates" by T. C. Lewis, 113–128.

MIND, Volume 44, July, 1919: "Professor John Cook Wilson" by H. A. Prichard, 297–318 ["The death of John Cook Wilson, Wykeham Professor of Logic in Oxford since 1889, is a serious loss for philosophy. . . . The following summary of his life is condensed from a notice by Mr. H. W. B. Joseph in Vol. VII of the *Proceedings of the British Academy*, to which the reader is also referred for a sketch of his philosophy." Professor Wilson died in August, 1915. He was the author of *On the traversing of geometrical figures*, Clarendon Press, 1905]; "On occupying Space" by H. W. B. Joseph, 336–339; "A proof that any aggregate can be well ordered" by P. E. B. Jourdain, 382–384 [First sentence: "The account of my process for well-ordering any given aggregate M described in *Mind* for July, 1918 . . . has been criticised by some on grounds which show, I think, that the point of the process has not been grasped."]

NOUVELLES ANNALES DE MATHÉMATIQUES, Volume 78, March, 1919: "Sur l'helicoëde gauche" by M. d'Ocagne, 81–89; "Sur une homographie particulière et son application a la perspective" by M. d'Ocagne, 89–92; "Sur une classe de courbes planes remarquables" by R. Goormaghtigh, 93–111; "Foyers rationnels des courbes" 112–113; "Applications géométriques de la théorie des infiniment petits" by M. d'Ocagne, 113–114; "Sur les courbes algébriques singulières sous le rapport des barycentres cycliques" by M. d'Ocagne, 115–116; Questions and solutions, 116–120—April: "Sur une application élémentaire d'une méthode générale donnant les équations du mouvement d'un système" by P. Appell, 121–131; "Sur les centres de courbure des lignes décrites par les points d'une figure plane mobile dans son plan" by M. d'Ocagne, 131–134; "Sur l'extraction, a une unité près, de la racine $m^{ième}$ d'un nombre quelconque a l'aide des logarithmes" by P. Deleus, 134–137; "Sur l'orthopole" by R. Goormaghtigh, 137–140; "Sur quelques intégrales trigonométriques" by M. F. Egan, 140–145; Questions and solutions, 145–160. —May: "Sur la méthode de Poincaré pour étudier la stabilité de l'équilibre" by P. Appell,

161–168; "Sur la détermination des surfaces minima" by C. Clapier, 169–176; "Sur un problème de géométrie infinitésimale" by R. Goormaghtigh, 177–184; "Sur un problème de mécanique" by E. Delassus, 184–188; problems and solutions, 188–200.

PSYCHOLOGICAL BULLETIN, Princeton, N. J., Volume 15, No. 12, December, 1918: "An easy method of determining the coefficient of correlation" by H. F. Adams, 456–459.

REVUE DE METAPHYSIQUE ET DE MORALE, Year 26, No. 3, May–June, 1919: "La question de la sincérité de Descartes" by G. Milhaud, 297–311; "Sur une définition possible des ordinaux transfinis" by A. Reymond, 313–334; "Sur la méthode d'enseignement des mathématiques et des sciences pour la formation du futur maître" by E. Rignano, 389–399.

REVUE DU MOIS, Tome 20, April, 1919: "L'orientation actuelle des mathématiques" by A. Denjoy, 18–28 [Discours d' installation dans la chaire de théorie des fonctions à l'université d'Utrecht, 3 Octobre, 1917].

REVUE GÉNÉRALE DES SCIENCES, Volume 30, No. 2, June 30, 1919: Review, by R. d'Adhémar, of Goursat's *Cours d'analyse mathématique*, tome 2, 3e éd. (Paris, 1918), p. 385; review by L. Potin of H. Bouasse's *Géographie mathématique* (Paris, 1919), 385–386.

SCIENCE, second series, Volume 50, August 8, 1919: "The problem of the boy in the swing" by H. Crew, 139–140.

SCIENCE PROGRESS, Volume 14, No. 53, July, 1919: "Recent advances in mathematics" by P. E. B. Jourdain, 1–10; "Recent advances in applied mathematics" by S. Brodetsky, 10–18; Essay-review, "The nature of number" by J. C. Gregory [on B. Russell's *Introduction to Mathematical Philosophy* (London, 1919)], 141–146; Reviews by P. E. B. Jourdain of A. Gray's *A Treatise on Gyrostatics and Rotational Motion: Theory and Applications* (London, 1918); of E. B. Elliott's *An Introduction to the Algebra of Quantics* (2d ed., Oxford, 1913); of H. B. Phillips's *Integral Calculus* (New York and London, 1917); of R. S. Heath's *Solid Geometry* (4th ed., London, 1919); and of W. P. Milne and G. J. B. Westcott's *A First Course in the Calculus* (London, 1918), 150–153.

TÔHOKU MATHEMATICAL JOURNAL, Volume 16, Nos. 1–2, June, 1919: "On measure of discontinuity and approximative representation of a function" by Y. Okada, 1–15; "On the approximation of a function of two variables by polynomials" by S. Takenaka, 16–25; "The popular books, Jinkôki and Kaisanki, in the Japanese mathematics" [in Japanese] by T. Hayashi, 26–40; "On some methods of construction in elementary geometry" by K. Yanagihara, 41–49 [section headings: 'Constructions with a ruler and compasses of given aperture'; 'Poncelet-Steiner's constructions'; 'constructions with a ruler of limited length and compasses of given aperture']; "Notes in elementary geometry" by T. Kubota, 50–52; "On the sum of two quadratic rests with respect to a prime ideal" by T. Matsumoto, 53–61; "On the theory of finite differences" by T. Kameda, 62–72; "A note on the convergence of the continued fraction with positive elements" by K. Shibata, 73–77; "On the mean center of the contact points of tangents to an algebraic curve and of tangent planes to an algebraic surface" by K. Shibata, 78–88; "Rational determination of the 'Wertigkeit' of algebraical correspondence on an algebraic curve" by M. Shibayama, 89–91; "Mathematical notes" by T. Kubota, 92–98; "On special systems of line equations having infinite unknowns" by K. Ogura, 99–102; "On the theory of approximating functions with applications to geometry, law of errors and conduction of heat" by K. Ogura, 103–154; "Some problems of circle and sphere divisions" by T. Hayashi, 155–159; "On the maximum modulus of norm-function" [in Japanese] by T. Hayashi, 160–171; New Books, Contents of Periodicals, Miscellaneous Notes, 172–183.

UNITED STATES NAVAL INSTITUTE PROCEEDINGS, Volume 45, July, 1919: "Great circle sailing—a few 'wrinkles' to save time" by H. G. S. Wallace, 1197–1199.—August: "Trajectories and their corrections" by A. G. Kirk, 1375–1395.

ZEITSCHRIFT FÜR MATHEMATISCHEN UND NATURWISSENSCHAFTLICHEN UNTERRICHT ALLER SCHULGATTUNGEN, Volume 50, No. 1, published Jan. 28, 1919: "Zum fünfzigsten Jahrgang der Zeitschrift" by W. Lietzmann, 1–7; "45 Jahre Aufgaben-Repertorium" by C. Müsebeck, 7–13; "Erinnerungen aus den ersten Jahren dieser Zeitschrift" by Hubert Müller, 13–18; "Die geometrische Periode in dem mathematischen Unterricht an der Breslauer Universität" by O. Toeplitz, 18–26; "Mathematische Wanderungen und Wandelungen in der Provinz Hessen-Nassau" by Carl H. Müller, 27–34; "Erinnerungen" by A. Schülke, 34–40; "Die Entwickelung des mathematischen Unterrichts im Sinne der Reformbewegung am Gymnasium in Göttingen'' by E. Götting, 40–47; "Winkelmessung durch Umlauf" by K. Schwering, 47–49; "Begabung und Studium" by W. Lorey [dedicated to F. Klein on the 50th anniversary

of his doctorate and in honor of his 70th birthday], 49–56; "Der mathematische Verein München" by H. Wieleitner, 56–58; "Emil Lampe zum Gedächtnis" [with portrait] by E. Haentzschel, 58–60; "Bücherbesprechungen," "Zeitschriftenschau," and "Neuerscheinungen," 62–64—No. 2, published Feb. 20: "Einteilung der Dreiflache" by R. Sturm, 65–66; "Ueber Winkelteilung" by R. Sturm, 66–69; "Versuch einer Bestimmung der mittleren Strafzeit" by R. Sturm, 70–71; "Zum geometrischen Anfangsunterricht" by G. Kewitsch, 71–78; "Winkel an den Parallelen" by Münch, 78–81; "Kleine Mitteilungen," 81–93; "Aufgaben-Repertorium," 93–97; "Bücherbesprechungen," 100–112—No. 3, published March 21: "Ueber das 'Linearzeichnen' an den höheren Schulen und sein Verhältnis zum 'Freihandzeichnen'" by F. Schilling, 113–131; "Die Kollineation in Prima" by A. Gerlach, 131–137; "Kleine Mitteilungen," "Bücherbesprechungen," "Zeitschriftenschau," and "Neuerscheinungen," 138–152.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2793. Proposed by J. L. RILEY, Stephenville, Texas.

If a, b, and c are complex and α, β, and γ real constants, the point

$$x = \frac{at^2 + 2bt + c}{\alpha t^2 + 2\beta t + \gamma}$$

traces a conic or a straight line when t takes all real values. *Stolz und Gmeiner.*

2794. Proposed by B. J. BROWN, Kansas City, Mo.

Find the value of $x^{e^x} + x^{x^e}$ when $x \doteq 0$ and when $x \doteq \infty$. *I. C. S.* 1902.

2795. Proposed by C. N. SCHMALL, New York City.

A square is described touching the ellipse $x^2/a^2 + y^2/b^2 = 1$, at the ends of its minor axis; a second ellipse is drawn circumscribing the square and tangent to the given ellipse at the ends of the major axis. The new ellipse is treated as the first and the process is continued until there are n new ellipses. Show that the last ellipse is a circle if the eccentricity of the original ellipse is $\sqrt{n/(n+1)}$.

2796. Proposed by N. P. PANDYA, Amreli, India.

Construct a triangle ABC having its centroid on a given ellipse, AB being a fixed diameter of the ellipse and C lying on one of the directrices.

2797. Proposed by E. J. OGLESBY, College of William and Mary.

Solve for x and y the simultaneous equations, $x^3 + y^3 = 35$ and $x^2 + y^2 = 13$.

2798. Proposed by S. A. COREY, Des Moines, Iowa.

Prove that every positive integer is equal to the sum of at most four squares.

SOLUTIONS OF PROBLEMS.

406 (Algebra) [March, 1914]. Proposed by S. A. COREY, Albia, Iowa.

Solve the system of equations:

$$(1-x)(a_1 + a_2y + a_3z) = d, \qquad (1-y)(b_1 + b_2x + b_3z) = g, \qquad (1-z)(c_1 + c_2x + c_3y) = h.$$

Discussion by H. S. Uhler, Yale University.

In order to introduce cyclic order we shall interchange two of the coefficients in the second equation and rewrite it as

$$(1 - y)(b_1 + b_2z + b_3x) = g.$$

Obviously, this modification will have no essential influence on the final result.

Solving the third equation for z we obtain

$$z = 1 - \frac{h}{c_1 + c_2x + c_3y}.$$

Substituting this expression for z in the first equation and performing elementary reductions, we find

$$\alpha y^2 + \beta y + \gamma = 0 \tag{1}$$

where

$$\alpha \equiv a_2c_3(1 - x),$$

$$\beta \equiv [c_3(a_1 + a_3) + a_2c_1 - c_3d] - [c_3(a_1 + a_3) + a_2(c_1 - c_2)]x - a_2c_3x^2,$$

$$\gamma \equiv [c_1(a_1 + a_3) - c_1d - a_3h] - [(c_1 - c_2)(a_1 + a_3) + c_2d - a_3h]x - c_2(a_1 + a_3)x^2.$$

The second equation (modified), when treated in the same manner, gives

$$Ay^2 + By + C = 0 \tag{2}$$

where

$$A \equiv c_3[(b_1 + b_2) + b_3x],$$

$$B \equiv [(c_1 - c_3)(b_1 + b_2) + c_3g - b_2h] + [c_2(b_1 + b_2) + b_3(c_1 - c_3)]x + b_3c_2x^2,$$

$$C \equiv -\{[c_1(b_1 + b_2) - c_1g - b_2h] + [c_2(b_1 + b_2) + b_3c_1 - c_2g]x + b_3c_2x^2\}.$$

Eliminating y from equations (1) and (2) we get

$$(\alpha C - \gamma A)^2 - (\alpha B - \beta A)(\beta C - \gamma B) = 0. \tag{3}$$

When the expressions for α, β, γ, A, B, C are substituted in relation (3) it is found that the coefficient of x^6 vanishes identically, but that all the remaining coefficients have extremely complicated values other than zero, in general. Therefore, since the equation in x is of the fifth degree with literal coefficients, the original set of equations cannot be "solved."

In the particular case where $a_1 = 2$, $a_2 = 3$, $a_3 = 4$, $b_1 = 5$, (new) $b_2 = 6$, (new) $b_3 = 7$, $c_1 = 8$, $c_2 = 9$, $c_3 = 10$, $d = 11$, $g = 12$, and $h = 13$, I found

$$1{,}628{,}991x^5 + 3{,}399{,}760x^4 - 3{,}151{,}759x^3 - 8{,}498{,}412x^2 - 8{,}179{,}912x - 8{,}133{,}008 = 0,$$

which has at least one positive real root.

411 (Algebra) [April, 1914, June, 1919]. Proposed by V. M. Spunar, Chicago, Ill.

Determine $x_1, x_2, x_3, \cdots x_p$, from the equations:

$$\begin{aligned}
x_1 + x_2 + x_3 + \cdots + x_p &= a_0,\\
b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_px_p &= a_1,\\
b_1^2x_1 + b_2^2x_2 + b_3^2x_3 + \cdots + b_p^2x_p &= a_2,\\
\cdots\cdots\cdots

$$D_2 = k\Delta(b_i - b_1)(b_i - b_2) \cdots (b_i - b_{i-1})(b_i - b_{i+1}) \cdots (b_i - b_p),$$

where $\Delta \equiv (b_1 - b_2) \cdots (b_1 - b_p)(b_2 - b_3) \cdots (b_2 - b_p) \cdots (b_{p-1} - b_p)$, and k can only be a non-literal factor. To determine k, we note that the product of all the first terms in the $\frac{1}{2}p(p-1)$ binomial factors equals $b_i^{p-1}b_1^{p-2}b_2^{p-3} \cdots b_{p-2}^2 b_{p-1}$ which differs only in sign from the algebraic value of the negative diagonal of D_2; hence, $k = -1$ or

$$D_2 = -\Delta(b_i - b_1)(b_i - b_2) \cdots (b_i - b_{i-1})(b_i - b_{i+1}) \cdots (b_i - b_p). \tag{1}$$

Next, consider D_1. If $a_0, a_1, a_2, \cdots, a_{p-2}, a_{p-1}$ were replaced, respectively, by $1, x, x^2, \cdots, x^{p-2}, x^{p-1}$ we should have (by the same argument)

$$D_x = -\Delta(x - b_1)(x - b_2) \cdots (x - b_{i-1})(x - b_{i+1}) \cdots (x - b_p)$$
$$\equiv -\Delta(x^{p-1} + c_1x^{p-2} + c_2x^{p-3} + \cdots + c_{p-2}x + c_{p-1}).$$

From the theory of equations, we know that $-c_1$ is the sum of $b_1, b_2, \cdots, b_{i-1}, b_{i+1}, \cdots, b_p$, that $+c_2$ is the sum of the products of these b's taken two at a time, etc., and that $\pm c_{p-1}$ is the product of all these b's, the upper or lower sign to be taken according as p is odd or even. Now the first minors that multiply the a's in D_2 are identical with the corresponding minors which multiply the various powers of x in D_x. Consequently,

$$D_1 = -\Delta(a_{p-1} + c_1a_{p-2} + c_2a_{p-3} + \cdots + c_{p-2}a_1 + c_{p-1}a_0). \tag{2}$$

Finally, dividing equation (1) by equation (2) we get the required formula

$$x_i = \frac{a_{p-1} - (\Sigma b)a_{p-2} + (\Sigma bb)a_{p-3} + \cdots \pm (\Pi b)a_0}{\Pi(b_i - b)},$$

the interpretation of the notation being clear from the preceding discussion.

Also solved by NORMAN ANNING.

455 (Geometry) [February, 1915; June, 1919]. Proposed by R. P. BAKER, University of Iowa.

Find the minimum triangle of assigned angles inscribed in a given triangle.

SOLUTION BY R. C. ARCHIBALD, Brown University.

This problem is closely allied to the problem: Find the maximum triangle of assigned angles circumscribed to a given triangle. Both problems were proposed for solution in 1811 on the last page (exclusive of index), 384, of the first volume of *Annales de mathématiques pures et appliquées*. Solutions by Rochat, and others, were given in volume 2, pages 88–93.

Let $D_1E_1F_1$ be the triangle of assigned angles and ABC be the given triangle. On CA (on the side opposite from B) describe an arc of a circle containing an angle equal to the angle E_1. On CB (on the side opposite from A) describe an arc of a circle containing an angle equal to D. Then if through C a line is drawn parallel to the line of centers of the circles it will meet the circles again in D and E such that DE is the greatest of all line-segments drawn through C. Then if EA, DB be produced to meet in F, the triangle DEF will clearly be the maximum triangle circumscribing the triangle ABC, and the triangle ABC will be the minimum triangle inscribed in the triangle DEF.

Hence to solve the given problem find the maximum triangle, $A_1B_1C_1$, similar to the triangle ABC and circumscribing the triangle $D_1E_1F_1$ (the point F_1 falling in A_1B, and the point E_1 in A_1C_1; also the angles A_1, B_1, C_1 correspond respectively to A, B, and C). Then divide AB at F in the ratio $A_1F_1 : F_1B$; also AC at E in the ratio $A_1E_1 : E_1C$; and CB at D as $C_1D_1 : D_1B_1$. Then D, E, and F are the vertices of the required minimum triangle.

In general, there are, as Rochat pointed out, six solutions of the problem, depending upon the arrangement of the vertices of the triangle DEF on the sides of the given triangle.

Another solution is given in *A Key to the Exercises in the First Six Books of Casey's Elements of Euclid* by Joseph B. Casey, second edition, 1887, pp. 120–121.

Note. A solution by John Casey is given in his *Sequel to . . . the Elements of Euclid*, second edition, 1882, pp. 38–39.—Editors.

Also solved by A. PELLETIER.

2737. Proposed by C. N. SCHMALL, New York City.

Employing Maclaurin's theorem, or otherwise, expand the following three functions (1) $e^{\tan^{-1} x}$ as far as x^6; (2) $e^{\sin x}$ as far as x^3; and (3) $\tan x$ as far as x^9.

The solution of this problem published in our issue for September was incorrectly ascribed to Elmer Latshaw; it was by G. A. OSBORNE, professor emeritus of the Massachusetts Institute of Technology—EDITORS.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

Mr. H. E. WOLFE has been appointed assistant professor of mathematics in Indiana University.

Dr. L. S. SHIVELY, who has been professor of mathematics in Mount Morris College, is now president of the college.

T. W. JACKSON, formerly principal of the high school in Fulton, Missouri, has been made head of the department of mathematics in Jamestown College.

H. L. OLSON has been elected to an instructorship in mathematics in the University of Wisconsin.

W. H. SHERK has been appointed professor of mathematics in the University of Buffalo.

Mr. NORMAN ANNING, recently discharged from the Canadian railway troops after service in France, has been appointed instructor in mathematics at the University of Maine.

Dr. H. C. GOSSARD, formerly instructor in the U. S. Naval Academy, has been made assistant professor of mathematics at the University of Oklahoma.

DR. H. C. WOLFF, formerly assistant professor of mathematics at the University of Wisconsin, has accepted an appointment as head of the department of mathematics at the Drexel Institute.

At Washington University, Professor GEORGE O. JAMES has been made head of the department of mathematics and Dr. J. R. MUSSELMAN, formerly of the University of Illinois, has accepted a position as instructor in mathematics.

Assistant professor GLENN JAMES, of Purdue University, has resigned to accept a similar position in the mathematics department at the Carnegie Institute of Technology.

The following men have been appointed instructors in mathematics at Purdue University: Messrs. R. S. UNDERWOOD, G. D. JAMES, F. H. HODGE and T. E. RAIFORD.

Dr. T. McN. SIMPSON has returned from Y. M. C. A. service in France and has been appointed professor of mathematics in his *alma mater*, Randolph-Macon College.

Mr. J. A. CAPARO, professor of electrical engineering and physics at the University of Notre Dame, received the A.M. degree in mathematics at the University of Chicago at the September convocation.

Mr. ELBERT ALLEN, formerly of the University of Chicago High School, has been appointed instructor in mathematics at the University of Missouri.

Dr. S. CHAPMAN, chief assistant at Greenwich Observatory, has been appointed professor of mathematics at the University of Manchester.

Professor W. H. YOUNG, formerly of the University of Liverpool, has been appointed professor of mathematics in the University College of Wales.

At Princeton, Professor PIERRE BOUTROUX and Assistant professor J. H. M. WEDDERBURN have returned after five years of absence in the French and British war service respectively.

Lieutenant F. L. CARMICHAEL, associate professor of mathematics at the University of Alabama, was in charge of range table calculations in the Ordnance Department at Washington during the past summer.

At the University of Illinois, Assistant Professor ARNOLD EMCH has been promoted to an associate professorship of mathematics and Mr. E. H. VANCE has been appointed instructor in mathematics.

Corrigenda: It should have been stated (*1919*, 419) that the value of the Querini-Stampalia prize was 3,000 *lire*—Mr. H. L. SMITH, of Cornell University, is instructor in mathematics at the University of Wisconsin and not as previously noted (1919, 322)—While it is true that Professor L. L. SILVERMAN resigned to enter a banking house in Boston (*1919*, 418) he later decided to remain at Dartmouth College.

THE WORK OF THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS, NOVEMBER, 1919.

Following a previous statement regarding the work of the National Committee on Mathematical Requirements (*1919*, 439), the following items may be of interest.

A preliminary report on "The Reorganization of Introductory Courses in Mathematics in Secondary Schools" was issued towards the end of November.

This report has been prepared by a representative sub-committee. It has not as yet been considered by the National Committee but its publication as a basis for discussion by teachers, organizations, committees, and local groups has been authorized.

A report on "The Valid Aims and Purposes of the Teaching of Mathematics in the Light of Recent Criticisms" will, it is hoped, be ready for distribution by January. In it an attempt will be made to state precisely and succinctly the mathematical training that every citizen should secure. The findings of this report can then be made a basis for the determination of precisely what and how much mathematics should be required of all students.

An extended investigation of "Experimental Schools and Courses" is being undertaken for the Committee by Mr. Raleigh Schorling of the Lincoln School. Detailed plans for this investigation were approved by the National Committee at its last meeting in New York City on November 1st.

Mr. J. A. Foberg is preparing a report on "Mathematics in Junior High Schools."

Professor A. R. Crathorne has recently submitted a report giving the results of his investigation of "Change of Mind Between High School and College as to Life Work." A summary of these results will be published in *School and Society* in the very near future. Professor Crathorne is still at work on an extended investigation entitled "A Critical Study of the Correlation Method as Applied to Grades."

A statement of general principles to govern the proposed revision of college entrance requirements has been tentatively approved by the Committee. This statement has been sent out to some 50 representative colleges and universities for their criticism and comment.

The Committee has sent letters to all teachers organizations having mathematical interests, of which it has been able to learn, asking their coöperation and offering the assistance of the Committee. The response has been very enthusiastic. A considerable number have already appointed committees to receive reports from the National Committee for study and criticism. The plans of the National Committee have been, or will be, presented to 22 such organizations during the months of October to December in the following states: Indiana, Illinois, Iowa, Kentucky, Massachusetts, Missouri, New Jersey, New York, North Carolina, North Dakota, Ohio, Oklahoma, Pennsylvania, Rhode Island, Texas, West Virginia and Wisconsin. Little has yet been done with reference to the promotion of new organizations. It was felt that this Committee should wait until it had definite material on hand for distribution before taking up this phase of its work. The formation of mathematical clubs in several of the larger cities, where they did not exist before, is, however, under way and material regarding the formation of such clubs is in hand. Any individuals interested in the formation of a mathematical club may secure this material by addressing the Chairman of the Committee (J. W. Young, Hanover, New Hampshire).

INDEX TO VOLUME XXVI.

Compiled by HELEN A. MERRILL and Department Editors.

Misprinted names in the text are corrected, and missing initials supplied, in this index.

FORMAL PAPERS—AUTHORS.

ADAMS, O. S. Meeting of the Maryland-Virginia-District of Columbia Section, 325.

ALTSHILLER, N. Remarks on a previous article, 65.

ARMSTRONG, G. N. Fourth annual meeting of the Ohio Section, 234.

BALLANTINE, J. P. Difference quotients, 53.

BARROW, D. F. On the envelope of the Wallace lines of an inscribed quadrangle, 108.

BARTON, R. M. Meeting of the Minnesota Section, 435.

BROGGI, UGO. A proof of a theorem of compound probabilities, 288.

CAIRNS, W. D. Election of members, 237.

——— Third annual meeting of the Mathematical Association of America, 91.

——— Fourth summer meeting of the Mathematical Association of America, 373.

CARMICHAEL, R. D. Fermat numbers $F_n = 2^{2^n} + 1$, 137.

CLAWSON, J. W. A theorem in the geometry of the triangle, 59.

COBLE, A. B. Concerning a method for finding a particular integral, 12.

DOWNING, H. H. Third annual meeting of the Kentucky Section, 279.

EMCH, A. On an elementary problem of closure on an equilateral hyperbola, 63.

FORD, W. B. (*Chairman*) Second report of the Committee on Libraries—list of college mathematical text-books, 225.

FRANKLIN, P. Some geometrical relations of the plane, sphere, and tetrahedron, 146.

FRUMVELLER, A. F. A theory and generalization of the circular and hyperbolic functions, 280.

HUNTINGTON, E. V. Mathematics and statistics with an elementary account of the correlation coefficient and the correlation ratio, 421.

LAVES, K. How the map-problem was solved in the war, 181.

LIGHT, G. H. The April meeting of the Rocky Mountain Section, 237.

LORIA, G. On certain constructions of descriptive geometry, 45.

MCCLENON, R. B. Leonardo of Pisa and his *Liber Quadratorum*, 1.

MACMILLAN, W. D. The growth of the solar system, 326.

MANNING, H. P. "Concerning a method for finding a particular integral," Note on Professor Coble's article, 113.

MILLER, G. A. Bits of history about two common mathematical terms, 290.

RIGGE, W. F. Cuspidal rosettes, 332.

ROEVER, W. H. Geometric explanation of a certain optical phenomenon, 111.

VIVIAN, ROXANA H. Statistics in relation to the war, 32.

WILCZYNSKI, E. J. On the form of the power series for an algebraic function, 9.

YOUNG, J. W. (*Chairman*) Report of the National Committee on Mathematical Requirements, 233, 279, 439, 462.

FORMAL PAPERS—TITLES.

Algebraic function, On the form of a power series for. E. J. WILCZYNSKI, 9.

Annual meeting, Third, of the Mathematical Association of America. W. D. CAIRNS, 91.

Circular and hyperbolic functions, A theory and generalization of. A. F. FRUMVELLER, 280.

Committee, National, on Mathematical Requirements. Report, 233, 279, 439, 462.

Committee on Libraries—List of college mathematical text books, 225.

Constructions of descriptive geometry, On certain. G. LORIA, 45.

Cuspidal rosettes. W. F. RIGGE, 332.

Descriptive geometry, On certain constructions of. G. LORIA, 45.

Difference quotients. J. P. BALLANTINE, 53.

Election of members. W. D. CAIRNS, 237.

Envelope of the Wallace lines of an inscribed quadrangle. D. F. BARROW, 108.

Fermat numbers $F_n = 2^{2^n} + 1$. R. D. CARMICHAEL, 137.

Geometric explanation of a certain optical phenomenon. W. H. ROEVER, 111.

Geometrical relations of the plane, sphere, and tetrahedron. P. FRANKLIN, 146.

Geometry, Descriptive, On certain constructiόns of. G. LORIA, 45.

Geometry of the triangle, A theorem in. J. W. CLAWSON, 59.

Growth of the solar system. W. D. MACMILLAN, 326.

History, Bits of, about two common mathematical terms. G. A. MILLER, 290.

Hyperbolic and circular functions, A theory and generalization of. A. F. FRUMVELLER, 280.

Hyperbola, equilateral, On an elementary problem of closure on. A. EMCH, 63.
I-Centres of a triangle, Remarks on a previous article. N. ALTSHILLER, 65.
Integral, Particular, Concerning a method for finding. A. B. COBLE, 12. Note on same, H. P. MANNING, 113.
Leonardo of Pisa and his *Liber Quadratorum*. R. B. MCCLENON, 1.
Libraries, Second report of Committee on. List of College Mathematical text-books, 225.
Map-Problem, How the — was solved in the war. K. LAVES, 181.
Mathematical Association of America. Third annual meeting of. W. D. CAIRNS, 91. Fourth summer meeting of, W. D. CAIRNS, 373.
Mathematical Association of America, Sections of. Kentucky, Third annual meeting. H. H. DOWNING, 279. Maryland-Virginia-District of Columbia, Meeting. O. S. ADAMS, 325. Minnesota, Meeting. R. M. BARTON, 435. Rocky Mountain, April meeting. G. H. LIGHT, 237.
Mathematical Requirements, Report of the National Committee on, 233, 279, 439, —.
Mathematical terms, two common, Bits of history about. G. A. MILLER, 290.
Mathematics and statistics. E. V. HUNTINGTON, 421.
Optical phenomenon, Geometric explanation of a certain. W. H. ROEVER, 111.
Plane, sphere and tetrahedron, Some geometrical relations of. P. FRANKLIN, 146.
Power series for an algebraic function, On the form of. E. J. WILCZYNSKI, 9.
Probabilities, compound, Proof of a theorem of. UGO BROGGI, 288.
Problem of closure on an equilateral hyperbola, On an elementary. A. EMCH, 63.
Quadrangle, inscribed, On the envelope of the Wallace lines of. D. F. BARROW, 108.
Report of Committee on Libraries, Second—List of college mathematical text-books, 225.
Report of National Committee on Mathematical Requirements, 233, 279, 439, —.
Rosettes, Cuspidal. W. F. RIGGE, 332.
Solar system, The growth of. W. D. MACMILLAN, 326.
Sphere, plane and tetrahedron, Some geometrical relations of. P. FRANKLIN, 146.
Statistics, Mathematics and. E. V. HUNTINGTON, 421.
Statistics in relation to the war. ROXANA H. VIVIAN, 32.
Tetrahedron, plane and sphere, Some geometrical relations of. P. FRANKLIN, 146.
Triangle, A theorem in the geometry of the. J. W. CLAWSON, 59.
Wallace lines of an inscribed quadrangle, On the envelope of the. D. F. BARROW, 108.
War, How the map-problem was solved in the. K. LAVES, 181.
War, Statistics in relation to the. ROXANA H. VIVIAN, 32.

QUESTIONS AND DISCUSSIONS—AUTHORS.

BENNETT, A. A. The sign of the distance in analytical geometry, 344.
BROWN, B. H. Probabilities in the game of "shooting craps," 351.
CARVER, W. B. Trigonometric functions—of what? 243.
ESCOTT, E. B. Reply to Question 21, 239.
EVANS, G. W. Concerning haversines in plane trigonometry, 69.
GILLESPIE, D. C. Advanced calculus or differential equations, 189.
HALDEMAN, C. B. Geometrical construction of the roots of a cubic, and inscription of a regular heptagon in a circle, 390.
HANCOCK, H. Reply to Question 36, 292.
JACOBS, M. W. An erroneous rule for finding a hypotenuse, with a corollary, 395.
JOHNSON, W. W. The complex quantity in elementary algebra, 295.
KAZARINOFF, D. C. Dupin's theorem, 441.
LIGHT, G. H. The existence of cusps on the evolute at points of maximum and minimum curvature on the base curve, 151.
LING, G. H. Reply to Question 32, 242.
MATHEWS, R. M. Graphical constructions for imaginary intersections of line and conic, 447.
MCCLENON, R. B. On the teaching of logarithms, 297.
MORITZ, R. E. On the quadrature of the parabola, 388.
NOBLE, C. A. The curve of concentration for a liquid mixture, 191.
RIETZ, H. L. On the teaching of the first course in calculus, 341.
RIGGE, W. F. Concerning a new method of tracing cardioids, 21.
SCHMIEDEL, O. On the summation of certain series, 392.
TREVOR, J. E. A property of homogeneous functions, 444.
WEAVER, W. Note on the teaching of the principle of mathematical induction, 190.
WHITTEMORE, J. K. Reply to Question 32, 188.

QUESTIONS AND DISCUSSIONS—SUBJECTS.

Calculus, Advanced, or differential equations. D. C. GILLESPIE, 189.
Calculus, Teaching of first course in. H. L. RIETZ, 341.
Cardioids, New method of tracing. W. F. RIGGE, 21.
Complex quantity in elementary algebra. W. W. JOHNSON, 295.

Concentration for liquid mixture, Curve of. C. A. NOBLE, 191.
"Craps," Probabilities in game of. B. H. BROWN, 351.
Cubic, Geometrical construction of roots of. C. B. HALDEMAN, 390.
Cusps on evolute at points of maximum and minimum curvature on base curve. G. H. LIGHT, 151.
Differential equations, Advanced calculus or. D. C. GILLESPIE, 189.
Distance in analytical geometry, Sign of. A. A. BENNETT, 344.
Dupin's theorem. D. C. KAZARINOFF, 441.
Editorial comment. U. G. Mitchell, 20–21; W. A. HURWITZ, 188–189, 241–243, 295, 340–341, 388, 441.
Evolute, Cusps on. G. H. LIGHT, 151.
Graphical constructions for imaginary intersections of line and conic, R. M. Mathews, 447.
Haversines in plane trigonometry. G. W. EVANS, 69.
Heptagon in a circle, Inscription of. C. B. HALDEMAN, 390.
Homogeneous functions, A property of. J. E. TREVOR, 444.
Hypotenuse, Erroneous rule for finding. M. W. JACOBS, 395.
Imaginary intersections of line and conic, Graphical constructions for. R. M. MATHEWS, 447.
Induction, Teaching of principle of mathematical. W. WEAVER, 190.
Liquid mixture, Curve of concentration for. C. A. NOBLE, 191.
Logarithms, Teaching of. R. B. MCCLENON, 297.
Mathematical induction, Teaching of principle of. W. WEAVER, 190.
Parabola, Quadrature of. R. E. MORITZ, 388.
Probabilities in the game of "shooting craps." B. H. BROWN, 351.
Quadrature of the parabola. R. E. MORITZ, 388.
Series, Summation of certain. O. SCHMIEDEL, 392.
Trigonometric functions—of what? W. B. CARVER, 243.

RECENT PUBLICATIONS—REVIEWS.

Archibald, R. C. *The Training of Teachers of Mathematics for the Secondary Schools of the Countries represented in the International Commission on the Teaching of Mathematics*, 67–68.
Barton, R. M. See Crawley, E. S.
Bell, R. J. T. See Frost, P.
Benedict, H. Y. See Karpinski, L. C.
Bennett, A. A. See Ford, L. R., and Milne, W. P.
Bianchi, L. *Lezioni sulla teoria dei gruppi continui finiti di transformazioni*. G. A. MILLER, 252–253.
Breckenridge, W. E. *Rehabilitation Monographs. Unit Course. Mathematics I. Use of the Slide Rule*, 358.
Cajori, F. See Child, J. M.
Calhoun, J. W. See Karpinski, L. C.
Carmichael, R. D. See Dickson, L. E.
Child, J. M. The Geometrical Lectures of Isaac Barrow. F. CAJORI, 15–20.
Colpitts, Julia T. See Roberts, Maria M.
Comstock, G. C. *The Sumner Line, or Line of Position, as an Aid to Navigation*. F. SLOCUM, 454.
Cornu, L. *Cours de mécanique*, tome 3, 118.
Crawley, E. S., and Evans, H. B. *Analytic Geometry*. R. M. BARTON, 113–116.
Cullis, C. E. *Matrices and Determinoids*, 202–203.
Currier, C. H. See Roberts, Maria M.
Dickson, L. E. *History of the Theory of Numbers*. Vol. I. *Divisibility and Primality*. R. D. CARMICHAEL, 396–403.
Dodd, E. L. See Whipple, G. C.
Dudeney, H. E. *Amusements in Mathematics*, 117–118.
Emch, A. See Veblen, O.
Encyclopædia of Religion and Ethics, 256.
Enriques, F. *Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche*, 204.
Evans, H. B. See Crawley, E. S.
Fischer, L. A. See Huntington, E. V.
Ford, L. R. *Elementary Mathematics for Field Artillery*. A. A. BENNETT, 353–355.
Forsyth, A. R. *A Treatise on Differential Equations; Solutions of the Examples in A Treatise on Differential Equations*, 157–158.
Frost, P., and Bell, R. J. T. *An Elementary Treatise on Curve Tracing*, 201–202.
Gill, J. *Text-book on Navigation and Nautical Astronomy*. Revised by W. V. Merrifield, 158.
Goodenough, G. A. See Waterbury, L. A.
Goursat, E. *Cours d'Analyse*, 255.
Halsey, F. A. *The Weights and Measures of Latin America*, 254–255.
Hedrick, H. B. *Interpolation Tables, or Multiplication Tables of Decimal Fractions*. D. N. LEHMER, 156–157.
Higbie, H. H. See Waterbury, L. A.
Huntington, E. V. and Fischer, L. A. *Handbook of Mathematics for Engineers*, 254.
Karpinski, L. C. *Four-place logarithmic and trigonometric tables*, 117.
——— See Kaye, G. R., and Sarkar, B. K.
Karpinski, L. C., Benedict, H. Y., and Calhoun, J. W. *Unified Mathematics*. G. W. MULLINS, 244–249.
Kaye, G. R. "The Astronomical Observatories of Jai Singh." L. C. KARPINSKI, 298–300.
Koren, J. *The History of Statistics*, 204.
Lehmer, D. N. See Hedrick, H. B.
Lennes, N. J. See Sutton, C. W.
Lipka, J. *Graphical and Mechanical Computation*, 203.

Lucas, A. *Des phénomènes gyroscopiques et de leurs principales applications à la navigation*, 357–358.
McClenon, R. B., and Rusk, W. J. *Introduction to the Elementary Functions*. G. W. MULLINS, 244–249.
——— See Nunn, T. P.
McKenzie, K. *The Italian Universities and their Opportunities for Foreign Students*, 300–301.
Merrifield, W. V. See Gill, J.
Miller, G. A. See Bianchi, L.
Milne, W. P., and Westcott, G. J. B. *A First Course in Calculus*. A. A. BENNETT, 355–357.
Morgan, F. M. *Essentials of Algebra and Geometry*, 301.
Mullins, G. W. See Karpinski, L. C., McClenon, R. B., Ransom, W. R., and Slichter, C. S.
New English Dictionary on Historical Principles, 256.
Nunn, T. P. *Exercises in Algebra (including Trigonometry)* and *The Teaching of Algebra (including Trigonometry)*. R. B. McCLENON, 154–156.
Poor, C. L. *Simplified Navigation for Ships and Aircraft*, 116–117.
Plummer, H. C. *An introductory treatise on Dynamical Astronomy*, 253–254.
Ransom, W. R. *Freshman Mathematics*. G. W. MULLINS, 244–249.
Roberts, Maria M., and Colpitts, Julia T. *Analytic Geometry*. C. H. CURRIER, 250–252.
Rusk, W. J. See McCLENON, R. B.
Sarkar, B. K. *Hindu Achievements in Exact Science*. L. C. Karpinski, 298–300.
Slichter, C. S. *Elementary Mathematical Analysis*. G. W. MULLINS, 244–249.
Slocum, F. See Comstock, G. C.
Smith, D. E. *Number Stories of Long Ago*, 301.
Sutton, C. W., and Lennes, N. J. *Business Arithmetic*, 159.
——— *Brief Business Arithmetic*, 159.
Veblen, O., and Young, J. W. *Projective Geometry*. A. EMCH, 194–201.
Waterbury, L. A., Goodenough, G. A., and Higbie, H. H. *A Vest-pocket Handbook of Mathematics for Engineers*, 158, 159.
Westcott, G. J. B. See Milne, W. P.
Whipple, G. C. *Vital Statistics. An Introduction to the Science of Demography*. E. L. DODD, 452.
Young, J. W. See Veblen, O.

RECENT PUBLICATIONS—NOTES.

[References to personal items published under this head are listed under "Personal Mention." References to other items may be found in the next section of this index.]

RECENT PUBLICATIONS—PERIODICALS.

Acta Mathematica, 404.
Alumni Register, University of Pennsylvania, 119.
American Journal of Mathematics, 161, 257, 404.
American Oxonian, 360.
Annaes Scientificos da Academia Polytechnica do Porto, 257.
Annali di Matematica pura ed applicata, 205.
Annals of Mathematics, 119, 258, 404.
Archivio di Storia della Scienza, 404.
Athenaeum, 258, 360.
Atlantic Monthly, 405.
Biometrika, 258.
Bollettino di Bibliografia e Storia delle Scienze Matematiche, 205, 360.
Bollettino di Matematica, 358.
Bulletin des Sciences Mathématiques, 360.
Bulletin of the American Mathematical Society, 120, 161, 303, 405.
Bureau of Education. Higher Education Circular, 276.
California Alumni Fortnightly, 255.
Columbia University Quarterly, 258.
Education, 162, 258.
L'Education Mathématique, 358.
Educational Administration and Supervision, 360.
Educational Review, 162, 405.
Elementary School Journal, 360.
Encyklopädie der mathematischen Wissenschaften, 160.
L'Enseignement Mathématique, 205–206, 360.
Harvard Graduates' Magazine, 405.
Hermathena, 360.
Industrial Arts Management, 456.
Isis, 118.
L'Intermédiaire des Mathématiciens, 206.
Jahrbuch ü. d. Fortschritte d. Mathematik, 403.
Jahresbericht der deutschen Mathematiker-Vereinigung, 162, 405.
Journal of Accountancy, 303.
Journal of Education, 258.
Journal of Educational Psychology, 303.
Journal of the Franklin Institute, 162.
Journal of the Engineers' Club of Philadelphia, 117.
Journal of the Indian Mathematical Society, 120, 206, 304, 406, 456.
Journal of the Michigan Schoolmaster's Club, 403.
Journal of Philosophy, Psychology, and Scientific Methods, 304.
Journal of the Washington Academy of Sciences, 258.
Matematisk Tidsskrift, 204, 258, 404.
Mathematical Gazette, 206, 304, 406, 456.
Mathematical Questions and Solutions . . . Educational Times, 204.
Mathematics Teacher, 120, 258, 406.
Mathematische Annalen, 403.
Memoirs of the National Academy of Sciences, 304.
Messenger of Mathematics, 120, 162, 360, 456.

Mind, 259, 456.
Monist, 120, 162, 305.
Monthly Weather Review, 162.
National Review, 259.
Nature, 120, 259, 361, 406.
Niew Archief voor Wiskunde, 305.
Nineteenth Century and After, 120.
Norsk Matematisk Tidsskrift, 455.
Notes and Queries, 305.
Nouvelles Annales de Mathématiques, 163, 206, 305, 361, 456.
Nyt Tidsskrift för Matematik, 120, 204, 260.
Open Court, 361.
Periodico di matematica per l'insegnamento secondario and *Supplemento*, 118.
Il Pitagora, 204.
Proceedings of the American Academy of Arts and Sciences, 120, 260.
Proceedings of the Edinburgh Mathematical Society, 120–121.
Proceedings of the London Mathematical Society, 163, 305.
Proceedings of the National Academy of Sciences of the U. S. A., 121, 163, 261, 406–407.
Proceedings of the Royal Society, 163, 407.
El Progreso Scientifico, 118.
Psychological Bulletin, 457.
Quarterly Journal of Pure and Applied Mathematics, 206–207, 407.
Record of the American Institute of Actuaries, 261.
Rendiconti del Circolo Matematico di Palermo, 204, 361.
Revista de la Sociedad Española, 302.
Revista de Matematicas, Buenos Aires, 121.
Revista matematica Hispano-Americana, 302.
Revue de Mathématiques Spéciales, 358.
Revue de Métaphysique et de Morale, 163, 207, 457.
Revue du Mois, 457.
Revue générale des Sciences, 207, 361, 457.
Revue philosophique de la France et de l'étranger, 163.
Revue scientifique, 361.
Rice Institute Pamphlets, 404.
School and Society, 163, 407.
School Review, 163, 407.
School Science and Mathematics, 163, 207, 261, 306, 361.
Science, 121, 164, 207, 261, 361, 407, 457.
Science Progress, 164, 302, 306–307, 457.
Scientia, 165, 207, 261, 307, 407–408.
Scientific American Supplement, 307.
Scientific Monthly, 165, 208.
Sewanee Review, 165, 208.
Sigma Xi Quarterly, 261.
Texas Mathematics Teachers' Bulletin, 121, 261, 307.
Tidsskrift för Matematik, 204.
Tidsskrift för elementar Matematik, Fysik, och Kemi, 361–362.
Tôhoku Mathematical Journal, 121–122, 165, 307, 408, 457.
Transactions of the Academy of Sciences of St. Louis, 208.
Transactions of the American Mathematical Society, 122, 208, 362.
Transactions of the Royal Society of Canada, 261, 408.
U. S. Naval Institute Proceedings, 457.
University Bulletin, Louisiana State University, 122.
Washington University Studies, 165.
Yale Alumni Weekly, 122.
Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht, 122, 165, 307, 457.

RECENT PUBLICATIONS—AMERICAN DOCTORAL DISSERTATIONS.

Benedict, Suzan R., 123.
Brand, L., 420.
Brink, R. W., 408.
Burgess, R. W., 123.
Campbell, J. W., 166.
Counts, G. S., 208.
Cox, L. C., 261.
Cresse, G. H., 420.
DePorté, J. V., 261.
Ford, L. R., 408.
Gossard, H. C., 123.
Grant, E. D., 362.
Griggs, Mary A., 123.
Harding, A. M., 166.
Hart, W. L., 166.
Haseman, Mary G., 166.
Hassler, J. O., 123.
Hazlett, Olive C., 123.
H'Doubler, F. T., 123.
Hollcroft, T. R., 261.
Howe, Anna M., 261.
Hu, M. T., 408.
James, G., 208.
Johnson, G., 123.
Kingston, H. R., 123.
Kline, J. R., 123.
Larew, Gillie A., 307.
McAtee, J. E., 362.
McMackin, F. J., 123.
Marriott, R. W., 123.
Matthewson, L. C., 123.
Merrill, A. S., 307.
Miller, Bessie I., 123.
Milne, W. E., 408.
Minnick, J. H., 261.
Miser, W. L., 123.
Monroe, W. S., 208.
Morrison, F. M., 166.
Moulton, E. J., 123.
Nelson, A. L., 123.
Paine, C. P., 87.
Reynolds, C. N., 420.
Rogers, Agnes L., 208.
Smith, D. M., 123.
Sperry, Pauline, 308.
Touton, F. C., 420.
Tsoo, Chan-Chan, 420.
Warren, L. A. H., 123.
Wells, Mary E., 166.
Wilder, C. E., 408.
Woody, C., 123.
Yeaton, C. H., 123.
Young, Mabel M., 123.

PROBLEMS—AUTHORS.

Numbers refer to pages, black-face type indicating a problem solved and solution published, italics a problem solved but solution not published, ordinary type a problem proposed.

Adams, O. S., 74.
Altshiller, N., 414.
Anning, N., 125, 271, 367, *460*.
Archibald, R. C., **416** (Note), **416**, **460**.
Arnold, Katherine S., *175*.
Baker, R. P., 268, 414.
Barnhart, C. A., *126*.
Bell, E. T., 269, 414, 416.
Bennett, A. A., 81, **415**.
Bramble, C. C., 312.
Brown, B. J., 129, 171, 414(2), 458.
Burgess, H. T., 213.
Cairns, W. D., 35, 268, *321*.
Campbell, A., 72.
Capron, P., 73, *126*, *129*, 171, *315*, *316*.
Carleton, H. E., *319*.
Carleton, H. N., *129*, *214*, *218*.
Carlson, H. E., *175*.
Chittenden, E. W., 35.
Clarke, E. H., 171, **177**, *273*, *313*.
Cohen, M., *320*.
Cohen, S., *126*.
Colwell, R. C., *175*, *314*, *316*.
Corey, S. A., 72, 79, 268, *313*, **315**, **369**, 458(2).
Cunha, P. J. da, *416*.
Daniell, P. J., **321**.
Davis, H. T., 125, 270.
Denton, W. W., 124.
Dickson, L. E., 214.
Dineen, A. T., *175*.
Dodd, E. L., 268.
Drane, W. H., 312.
Dunkel, O., 415, **416** (Remarks).
Echols, W. H., 366.
Escott, E. B., 37.
Fajans, I. J., *126*.
Fazio, J. V., *127*.
Flanagan, C. E., *129*, 312.
Franklin, P., 127.
Gilpin, C., Jr., 414.
Githens, C. E., **126**.
Gladstone, H., *127*.
Gossard, H. C., **128**.
Grant, E. D., *321*.
Greenstreet, W. J., 213.
Gronwall, T. H., 213, 366.
Gummer, C. F., **74**, **76**, **78**, **82**, **125**, **128**, **173**, **174**, **177**, **271**, *273*, **367**, **369**.
Hansen, P., **175**.
Harding, A. M., 73, 171(2), **272**, **273**, *313*(2), *314*, **317**, *317*, *319*, *370*, *416*.
Harlow, W. F., 174.
Hitt, J. R., 128.
Hoover, W., **79**, **129**(2), *131*, **314**, **316**, *317*, **319**.
Hopkins, G. I., 37.
Howard, H. R., **318**.
Huntington, E. V., 172, 173, **273**.
Irwin, F., 212, **269**, **270**, **271**, **367**, 369.
Johnson, R. A., *126*, **131**, *175*, *218*, *321*, 366.
Johnson, W. W., 171.
Julian, J. J., *175*.
Kempner, A. J., 269.
Kingston, H. R., *318*.
Kreth, D., 72.
Landis, W. W., 214.
Lane, E. P., 124.
Langman, H., 171.
Lasley, J. W., Jr., 73, 126, *127*, 414.
Latshaw, E., 311; see also Osborne, G. A.
Laves, K., 414.
Lawrence, J. F., 213.
Lehmer, D. N., **365** (Note).
Lewis, Florence P., 80.
Lochner, R. P., 312.
Lord, W. L., *127*.
Loud, F. H., *218*.
Lunn, L. E., 368.
Macaulay, C., 214.
McCain, Gertrude I., *370*.
Martin, A., 214, 215, 269, 416.
Mathews, R. M., *127*, *175*.
Mensenkamp, L. E., *126*, **127**, **315**.
Millenky, I., *126*.
Mills, C. N., 126, 319.
Moore, R. E., 72, 177, *312*.
Moots, E. E., 416.
Moritz, R. E., 414.
Morris, F. R., 76.
Nauer, A. N., *129*, *317*, **368**.
Oglesby, E. J., 458.
Olson, H. L., 35, *127*, *129*, *131*, **131**, *175*, *178*, **313**, *317*, *318*, **319**, *319*, *320*, *321*(2).
Osborne, G. A., **320** (incorrectly credited to E. Latshaw, 461).
O'Shaughnessy, L., 37.
Paaswell, G., *129*, 171.
Pandya, N. P., *127*, 131, 170, 171, 313, 369, 458.
Pelletier, A., **312**, *318*, *319*(2), *321*, *416*, *460*.
Phillips, H. B., 73, **320**.
Platman, A., *127*.
Post, E. L., **37** (Discussion).
Pumo, P., *127*.
Ransom, W. R., *321*.
Rees, E. L., **214**.
Reynolds, J. B., 35, 72, **75**, **77**, **78**, **80**(2), **81**, *124*, **126**, *127*, *129*, 129, **172**, **173**, *315*, *316*, *319*.
Richardson, L., 124.
Riley, J. L., 73, 124, *127*, 173, 268, 311, **320**, *321*, 458.
Roeser, H. M., **176**.
Rose, I., *127*.
Rosenbaum, J., 212, *321*.
Safford, F. H., *313*, **415**.
Schmall, C. N., 35, 36, 72, 124, 170, 213, 269, *320*, 458, 461.
Schweitzer, A. R., 312.
Shidy, L. P., *218*.
Simonsen, O. E., *126*.
Smith, J. B., 268.
Sousley, C. P., 267, *320*.
Spunar, V. M., 213, 268, 415, 459.
Swift, E., **124**, **125**, *127*, *129*, *321*, **369**, *416*.
Thompson, H. D., **129**.

Thomson, T. R., *127*.
Uhler, H. S., 36, 175, **215**, **459** (Discussion), **459**.
Weaver, W., 175, 268, 311, 366, 413.
Weinberger, S., *127*.
Weisner, L., **73**, **74**, **314**.
Whittemore, J. K., *369*.
Wildermuth, R. B., *175*.
Wilson, A. H., **317**.
Witmer, E. E., *313*, *318*.
Witmer, E. W., 72.
Worthington, E. H., 272, **319**.
Yanney, B. F., *218*.
Yen, C. C., 127.

PROBLEMS—SOLUTIONS.

[Numbers in black-face type refer to problems, those in light-face to pages.]

Algebra, **406**, 459 (Discussion); **411**, 459; **430**, 415.
Geometry, **455**, 460; **519**, 415, 416 (Remarks).
Calculus, **385**, 73; **411**, 124; **425**, 74; **433**, 37 (Discussion); **440**, 74.
Mechanics, **322**, 76; **328**, 78; **332**, 416 (Correction); **349**, 79; **350**, 80; **356**, 80; **357**, 81.
Number Theory, **198**, 416 (Note); **201**, 416; **231**, 269; **239**, 125, 270; **245**, 367; **247**, 271; **260**, 81; **261**, 125.
2665, 126; **2667**, 214, 315; **2670**, 126; **2678**, 271; **2679**, 126; **2680**, 127; **2681**, 127; **2682**, 214; **2683**, 128; **2684**, 129; **2685**, 129; **2687**, 131; **2689**, 172; **2690**, 173; **2691**, 131; **2692**, 173; **2693**, 174; **2696**, 368; **2697**, 175; **2698**, 175; **2699**, 177, 312; **2700**, 215, 365 (Note); **2701**, 272; **2702**, 313; **2703**, 313; **2704**, 314; **2705**, 315; **2707**, 315; **2708**, 316; **2709**, 273; **2710**, 312; **2711**, 316; **2712**, 317; **2714**, 318; **2715**, 318; **2716**, 319; **2722**, 369; **2729**, 369; **2731**, 369; **2735**, 319; **2736**, 320; **2737**, 320, 461; **2738**, 321. List of unsolved problems, 213.

MATHEMATICAL CLUBS—ACTIVITIES.

Albion College, 408.
Barnard College, 308.
Brown University, 167.
Columbia University, 262.
Denison University, 308, 362.
Goucher College, 365.
Harvard University, 262.
Hunter College, 168.
Northwestern University, 363.
Swarthmore College, 168.
University of Illinois, 309.
University of Kansas, 208.
University of Kentucky, 309.
University of Maine, 263.
University of Minnesota, 209.
University of Nebraska, 263.
University of Texas, 364.
University of Toronto, 169.
University of Washington, 170.
Vassar College, 264.

MATHEMATICAL CLUBS—TOPICS FOR PROGRAMS.

The Number π, 209. Codes and Ciphers, 409.
For social meetings:
An Evening at Chess, 311—Flatlanders—A Mathematical Play in One Act, 264—Mathematical Contests, 310—Mathematical Plays and Games, 310, footnote—Telling Fortunes by Mathematics, 310.

PERSONAL MENTION.

Adams, C. R., 89, 167, 274, 386; Adams, E. P., 324; Adams, H., 359; Adams, H. F., 457; Adams, O. S., 40, 325; Adkins, L. K., 40; Agnew, P. G., 276; Ahrens, W., 403; Alasia, C., 323; Aldrich, Maelynette, 386; Alexander, D., 409; Alexander, A., 324; Alexander, J. W., 40; Alger, P. L., 101; Allen, E., 462; Allen, E. S., 277, 419; Allen, J., 179, 221; Allin, C. S., 40; Allingham, W., 118; Altshiller, N., 119; Amaldi, U., 371; Ames, J. S., 276; Ames, W. L., 67; Amig, Margaret, 365; Amerman, C., 359; Anderson, W. E., 235; Anning, N. H., 40, 461; Antoine, —., 371; Apple, A. T. G., 105; Archibald, R. C., 88, 90, 104, 120, 133, 277, 380; Armstrong, L. E., 87; Arnaud, J. J., 40, 103; Arnold, C. L., 363; Ashton, C. H., 180, 208; Atchison, C. S., 40, 83; Autenrieth, G. C., 103.
Babbitt, A., 263; Babcock, R. W., 180; Babcock, Wealthy, 208; Bachman, P., 161; Bacon, Clara L., 236, 365; Bacon, R., 259; Baker, A., 169; Baker, R. P., 93, 94; Baldwin, Evelyn, 308; Baldwin, J. W., 85; Ballantine, S., 386; Ballard, Alice, 364; Ballore, R. M. de, 358; Barbarin, P., 223; Barnard, R. W., 40, 88; Barnett, I. A., 275; Barney, Ida, 322; Barrow, D. F., 40; Barth, C. G., 117; Barthelmy, A., 207; Barton, Helen, 219; Barton, R. M., 420, 438; Bass, E. W., 84; Batchelder, P. M., 121, 307; Batchelor, G., 405; Bateman, H., 120, 303, 456; Bauer, G. N., 275; Bauer, L. A., 324, 375; Beal, F. W., 370; Beal, W. O., 435; Beall, Mary E., 309; Beall, Sarah, 386; Beatley, R., 40; Becker, K., 159; Beckwith, Mrs. E. R., 133; Beckwith, W. S., 238; Beetle, R. D., 417; Belcher, D. R., 40; Bell, E. T., 170, 303; Bell, Rachel, 208; Belser, F. C., 303; Beman, W. W., 224, 276, 374, 420; Benedict, H. Y., 66, 89, 222, 261; Bennett, A. A., 40, 89, 98, 307, 364, 405, 418;

Benteli, A., 179; Berkele, Edna, 309; Bernstein, B. A., 133, 162; Berry, W. J., 40; Betz, H., 40, 85, 370; Betz, W., 277; Biefeld, P., 308; Biehl, Louise, 168; Biel, E. G., 40, 86, 322; Birchenough, H., 277; Birkhoff, G. D., 90, 162, 222, 275, 301, 324, 419; Black, Florence L., 386; Blair, Vevia, 277, 280; Blakslee, T. M., 386; Blanchard, R. L., 40; Blau, R. S., 168, 169; Bledsoe, J. M., 121; Blichfeldt, H. F., 40, 89, 98, 106, 119, 132, 219, 223, 276, 405; Bliss, G. A., 40, 89, 180, 224, 420; Blumberg, H., 105, 120, 208, 303, 404, 419; Bôcher, M., 136, 262, 372; Bohannon, R. D., 133; Bohr, H., 204; Bolton, Mrs. Helen, 261; Bond, J. D., 417; Bonnesen, T., 90, 204; Booth, A. L., 406; Borel, E., 404; Botkin, Frances R., 263; Bottaso, M., 372; Boussinesq, V. J., 419; Bouton, C. L., 222, 277; Boutroux, P., 40, 462; Bower, Ethelwyn, 168, 169; Bowes, R., 219; Boyack, Breeze, 237; Boyd, P. P., 279, 309, 324; Boyer, P. A., 258; Brackett, F. P., 133; Bradley, H. C., 275; Bramble, C. C., 161; Brand, L., 420; Brand, C. E., 307; Brandeberry, J. B., 40; Brandenburger, C., 372; Brasch, F. E., 407; Bray, H. E., 258; Breckenridge, W. E., 120, 223, 277; Breed, S. A., 275; Breslich, E. R., 359; Brill, A. V., 86; Brink, R. W., 209, 275; Broadlick, J. N., 40; Brockett, P., 159–160; Brooks, J., 264; Brower, Mary, 365; Brown, B. H., 40, 238, 275; Brown, B. J., 169; Brown, E. W., 90, 122, 220, 277, 324, 375, 380, 405; Brown, F. L., 417; Brown, Katharine D., 104; Brown, Marie, 209; Brown, T. H., 167; Browne, E. T., 417; Buchanan, D., 257; Buchanan, H. E., 105, 179, 221; Buck, T., 40, 89; Bullard, J. A., 219; Bumstead, H. A., 88; Burger, W. H., 364; Burgess, H. T., 40; Burgess, R. W., 40, 274, 360; Burnside, W., 220; Burton, E. F., 169; Burwell, W. R., 89; Bussey, W. H., 209, 438; Butler, P., 104; Butterfield, K. L., 220; Byerly, W. E., 262, 263.

Cairns, W. D., 93, 94, 180; Cajori, F., 90, 121, 133, 160, 164, 359, 404; Caldwell, —., 221; Caldwell, Minnie W., 104; Calhoun, J. W., 66, 261, 364; Callaway, Mrs. Theodosia T., 238; Camp, C. C., 40, 274; Campbell, A. D., 40, 87, 134, 275, 419; Campbell, J., 409; Campbell, Jessie R., 238; Campbell, J. W., 40; Canaday, E. F., 41; Cantor, M., 323; Caparo, J. A., 41, 462; Capron, P., 219; Carathéodory, C., 85; Card, H. L., 41; Carey, H. F., 167; Carmichael, F. L., 41, 462; Carmichael, R. D., 92, 104, 120, 121, 161, 162, 165, 303; Carrol, Ethel, 365; Carter, C. C., 104; Carty, J. J., 88; Carus, P., 135; Carver, W. B., 221; Castor, Mary E., 41; Cattell, J. McK., 403; Cawl, F. R., 407; Chace, A. B., 167; Chapman, S., —; Charles, R. L., 417; Chazy, J., 371; Child, J. M., 302; Child, Mabel C., 169; Chittenden, E. W., 133, 362; Clairin, J., 371; Clark, Lena, 168; Clarke, Helen, 308; Clawson, J. W., 404; Clements, G. R., 219; Cobb, C. W., 41; Coble, A. B., 161, 180; Codd, A. W., 135; Coe, C. J., 276; Coffin, J. G., 386; Cohen, A., 222, 325; Cohen, L., 276; Cohen, M. P., 304; Cohen, Teresa, 404; Cohn, A., 405; Cole, F. N., 89, 90, 263, 304, 405; Collier, Myrtle, 372; Colpitts, Mary A., 419; Colwell, R. C., 120; Conner, J. F., 41; Coolidge, J. L., 41, 220, 275, 361; Cornell, T. A., 89; Cornu, L., 118; Corson, E. T., 168; Courtright, Margaret, 409; Craig, C. E., 221; Craig, Jessie, 209; Crandall, C. L., 205; Cranz, C., 159; Crathorne, A. R., 93, 94, 279, 359, 439, 463; Cresse, G. H., 178, 276, 420; Cressy, D. H., 41; Crew, H., 457; Crooks, C. G., 279; Crum, W. L., 41; Cullis, C. E., 135; Cummings, Louise D., 303, 304; Currier, C. H., 41, 133; Curtis, H. B., 418; Curtis, Mary F., 119; Curtiss, D. R., 93, 95, 364, 420.

Daniell, P. J., 122, 303, 404; Daniells, M. E., 418; Daniels, F., 179; Danielson, F., 363; Dantzig, T., 178; Dappert, J. W., 105; Darboux, G., 371; Darmois, G., 371; Darwin, G. H., 179; Datta, B., 257; Davis, C. P., 262; Davis, E. W., 105; Davis, H. N., 275; Davis, J. E., 41; Davis, J. M., 279, 310; Davis, N. F., 167; Davis, Varina, 365; De Land, T. L., 307; Dean, W. C., 408, 409; Dearle, R. C., 169; Decherd, M. E., 222; Decker, F. F., 120, 418; Deering, Edith I., 263; De Long, I. M., 237; Deputy, Helen, 169; Dickson, L. E., 90, 92, 132, 223, 255, 258, 277, 303, 380, 405; Dillingham, A., 219, 325; Dillon, —., 221; Dines, C. R., 41, 89, 417; Dines, L. L., 84, 164, 208, 258; Dingeldey, F., 160; Dini, U., 205, 455; Dobson, W. P., 238; Dodd, E. L., 105, 222; Dodderidge, Faye, 208; Doggett, L. A., 325; Doll, T., 41, 87, 363, 418; Doolittle, C. L., 178; Dotterer, R. H., 304; Dougherty, H. R., 41; Dougherty, Lucy T., 41; Douglass, R., 275; Dowling, L. W., 180, 303, 405; Downey, W., 439; Downing, H. H., 279, 309; Dresden, A., 41, 303, 417; Doyle, Irene, 309; Dugan, R. S., 256; Duncan, F., 84; Dunkel, O., 105, 163, 165, 370; Dupuis, N. F., 205; Durand, W. F., 88.

Easterbrooks, Rachel T., 167; Edington, W. E., 41, 134; Eisenhart, L. P., 106, 361, 374, 380, 404; Ellery, Louise, 365; Elliott, W. W., 134, 279, 309; Ellis, D. S., 41; Emch, A., 121, 161, 303, 307, 407, 462; Engle, Viola, 208, 209; Epperson, C. A., 41; Eppes, J. B., 219; Erskine, J., 220; Esclangon, E., 371; Escott, E. B., 261; Eshleman, J. D., 41, 88; Ettlinger, H. J., 41, 222; Etzel, W. E., 436; Evans, G. C., 41, 178, 405; Evans, G. W., 120, 280, 439; Evans, H. B., 119; Everett, H. S., 104.

Fabey, L., 223; Fabre, J. H., 161; Fales, E. N., 159; Farris, —., 41; Fentress, G. L., 221; Ferguson, W. J., 104; Ferguson, Z., 84, 135; Ferrier, T. A., 135; Ferry, F. C., 277; Fetter, F., 169; Fiedler, W., 160; Field, P., 41; Fields, J. C., 169; Fine, H. B., 278; Finkel, B. F., 105, 106; Fischer, C. A., 223, 322; Fisher, W., 261; Fiske, T. S., 136; Fite, W. B., 88, 122, 222, 223; Flanagan, C. E., 105; Fletcher, W. H., 407; Flexner, S., 88; Floody, Lily M., 169; Floquet, A. M. G., 419; Foberg, J. A., 279, 439, 463; Foetz, L. S., 237; Foord-Kelcey, J. W., 135; Foraker, F. A., 406; Ford, L. R.,

41, 88; Ford, W. B., 93, 95, 224, 359, 375, 383; Forsyth, C. H., 41; Foster, Alice W., 169; Foster, J. M., 386; Fox, Ethel, 365; Fox, M., 408; Fox, P. J., 121; Fraleigh, E. P., 84; Fraleigh, P. A., 41, 135, 218; Franklin, P., 41, 238; Franklin, W. S., 162, 255, 276; Frary, H. D., 41; Freas, Elizabeth, 238; Fréchet, R. M., 178, 371; Free, Lucile, 264; Fricke, R., 403; Frizell, A. B., 89; Frobenius, G., 85; Fry, T. C., 41; Fu, M. F., 122; Furman, F. De R., 87.

Gaba, M. G., 84, 263, 370; Gaehr, P. F., 261; Galajikian, H., 41, 104; Galdeano, Z. G. de, 118, 302; Galle, A., 161; Galloway, J. N., 219; Garnet, W. H., 134; Garrett, W. H., 219; Garretson, W. V. N., 161, 417; Garrison, D. M., 221; Gauss, C. F., 161, 358; Gavin, Dorothy G., 169; Gaylord, H. D., 258; Gehman, H. N., 41, 104; Georges, J. S., 238; Gibbens, Gladys, 274; Giles, Ila B., 169; Gill, B. P., 41; Gillespie, D. C., 221, 258; Gilman, R. E., 41, 274; Gingrich, C. H., 435, 438; Glasgow, Josephine, 84; Gleason, R. E., 133; Glenn, O. E., 120, 261, 362; Glover, J. W., 419; Godfrey, S. C., 41; Goff, R. R., 258, 261; Golden, M. J., 87; Gordon, Elizabeth, 409; Gossard, H. C., 461; Gould, Alice B., 99, 104; Graber, M. E., 235, 418; Graf, J. H., 86; Graham, Marion, 168; Graustein, W. C., 89, 219; Gray, Nora E., 169; Green, C. F., 41; Green, G. M., 86, 121, 362; Green, J. L., 120; Grennan, Elizabeth B., 84; Groff, Deldee, 365; Gronwall, T. H., 42, 89, 120, 121, 258, 278, 404, 405, 407, 418; Gross, W., 178; Grossman, A., 84; Grove, V. G., 274; Guichard, C., 371; Guimarães, R., 371; Guy, D. J., 42.

Haag, J., 371; Hadamard, J., 220, 302; Hadaway, Joyce, 409; Hadley, L., 418; Hagen, Beatrice, 209; Hale, G. E., 88; Hall, E. O., 169; Hall, E. H., 120; Hallett, G. H., 276, 303; Hancock, H., 161; Harding, A. M., 261, 408; Harmon, A., 370; Harnly, P. W., 42; Harper, F., 409; Harris, R. A., 105; Hart, J. N., 263; Hart, W. L., 42, 83, 221, 275; Hart, W. W., 180, 359, 387; Haskell, Marian, 308; Haskins, C. N., 89; Hassé, D. H. R., 371; Hathaway, A. S., 407; Hawkes, H. E., 90, 277, 359; Hawkesworth, A. S., 42; Hayashi, T., 120, 303; Hazlett, Olive C., 122; Heal, W. E., 42; Heath, T. L., 302; Hebbert, C. M., 42; Hedlund, M., 42; Hedrick, E. R., 84, 92, 134, 220, 374, 379; Heegaard, P., 88, 204, 324; Hemke, P. E., 87, 370; Henderson, A., 165, 208; Henry, Phillis, 307; Hering, C., 162; Herrick, D., 409; Hildebrandt, T. H., 42, 88, 180; Hill, L. S., 42; Hill, P. W., 42, 322; Hjelmslev, J., 159; Hodgdon, Grace H., 263; Hodge, F. H., 462; Hölder, O., 85; Hollcroft, T. R., 42, 84, 88, 257; Hollcroft, Mrs., 84; Holtwick, Adele C., 238; Hooker, J. D., 302–303; Hopkins, L. A., 374; Hornung, C., 105; Horton, Goldie P., 119, 222, 307, 364; Howe, Anna M., 257; Howe, R. H., 309, 363; Howie, J. M., 42; Hubble, E. P., 42; Huney, Lois, 209; Huntington, E. V., 42, 92, 106, 375, 383, 418; Hurwitz, W. A., 90, 105, 221; Hussey, W. J., 324; Huxley, L., 302.

Ingels, Nelle L., 406; Ingold, L., 160; Irwin, H. N., 163; Isaacs, C. A., 387.

Jackson, D., 42, 159, 275, 278, 361; Jackson, T. W., 461; James, G., 461; James, G. D., 462; James, G. O., 461; Janes, L., 364; Janes, W., 364; Jeans, J. H., 160, 302; Jensen, J. L. W. V., 223; Jentz, Veronica, 308; Jentzch, R., 179; Jessen, I. L. W., 204; Joffe, S. A., 407; Johnson, Alice, 308; Johnson, Dorothy, 168; Johnson, M. F., 387, 418; Johnson, R. A., 435–436, 438; Johnson, W. W., 325, 361; Jones, A. L., 277; Jones, A. T., 407; Jones, E. L., 407; Julia, G., 223; Junkin, Vera, 409.

Karapetoff, V., 407; Karpinski, L. C., 66, 117, 163, 169, 224, 276, 370, 378, 407, 420; Kasner, E., 222, 308; Kauffman, H. N., 164; Kazarinoff, D. C., 420, 436; Keachie, F. B., 169; Keffer, R., 42; Kellogg, O. D., 42, 218, 323; Kells, L. M., 42; Kelly, Helen, 363; Kendall, Claribel, 222; Kennard, Elaine, 308; Kennedy, Carla, 409; Kennelly, A. E., 260; Kenyon, A. M., 42, 93, 160; Keppel, H. G., 84, 88, 105; Kerr, F. L., 221, 370; Keyser, C. J., 162, 258, 262, 304, 308; Kiefer, E. C., 387; Kiernan, A., 219; Killam, S. D., 42; Kimberly, M. C., 263; Kimbrough, Frances, 310; Kimmel, H., 163; King, E., 308; King, W. J., 219; Kingston, H. R., 83; Kinzel, A. B., 262; Kircher, E. A., 42, 135, 275, 418; Kirchner, W. H., 435; Kline, J. R., 258, 322; Klopsteg, P. E., 407; Klotz, O. J., 374; Knabelman, M. S., 322; Koenigs, G. X. P., 134; Koenigsberger, L., 86, 323; Kohs, S. C., 163; Kuhn, H. W., 133.

La Master, C. E., 261; Lamb, H., 302; Lambert, W. D., 42; Lampe, K. O. E., 89; Lampen, A. E., 104; Lamson, K. W., 223; Landis, W. W., 42; Lane, E. P., 104; Larew, Gillie A., 208; Larmor, J., 223; Lattès, S. I., 90, 223, 371; Laves, K., 98; Leaute, H. C. V. J., 134; Lefschetz, S., 261, 407; Lehmer, D. N., 105, 121, 163, 261; Lehr, Marguerite, 365; Lester, O. C., 237; Leuschner, A. O., 42; Leut, Ellen, 308; Levi, Mattie, 169; Levi-Civita, T., 371; Lewis, Florence P., 277, 365; Liapounoff, A. M., 179; Light, G. H., 222, 274; Lillard, R. W., 120; Lindsay, R. B., 167; Linebarger, C. E., 403; Ling, G. H., 418; Lipka, J., 277; Little, A. S., 303; Livingston, G. R., 387; Logsdon, Mrs. Mayme I., 274, 363; Longley, W. R., 42, 87; Lorentz, H. A., 135; Loria, G., 404; Lotka, A. J., 258; Love, A. E. H., 302; Lovejoy, A. O., 89; Lowe, P., 169; Luby, W. A., 359; Luck, J. J., 85; Lunn, A. C., 90, 383; Lytle, E. B., 384.

MacFarlane, A., 255; MacKavanaugh, T. J., 325; Mackay, Mary I., 169; Maclay, J., 222, 223, 262; MacLean, N. B., 42, 83; MacMillan, W. D., 42, 87, 101, 180, 370; MacNeish, H. F., 163; MacNutt, B., 255; Maennchen, P., 358; Manning, H. P., 105; Manning, W. A., 88, 208; Mansion, P., 323; Markham, E., 165; Markley, J. L., 276, 374; Marm, Anna, 387; Marshall, W., 42; Martin, A., 83, 105; Martin, E. W., 387; Mason, M., 42; Massey, R., 238; Matheson, J., 378; Mathews, R. M., 163, 435, 438; Mathewson, L. C., 122; Matos, W. A., 169; Maxwell, J. C., 220; McAtee, J. E., 87, 105, 303, 404; McClenon, R. B., 67, 105;

McCrae, J. D., 120; McCue, M. J., 104; McDonald, B., 364; McDowell, Carroll, 209; McKelvey, J. V., 42, 370; McMartin, Christine, 364; McSorley, Kathryn, 163, 168; Meneely, Janet, 308; Menk, Laura, 209; Mensenkamp, L. E., 163; Merriam, Frances M., 167; Merrill, A. S., 42, 257; Merrill, Helen A., 105, 120, 464; Merrill, P. W., 276; Metcalf, J. H., 405; Metcalf, W. V., 42; Metzler, W. H., 120, 121, 179; Meyer, W. F., 160; Mikesh, J. S., 42, 134; Milhaud, G., 207; Millay, Kathleen, 264; Miller, C. W., 42; Miller, F. E., 275; Miller, G. A., 88, 90, 121, 122, 163, 164, 207, 255, 257, 261, 277, 302, 303, 306, 360, 379, 404, 406, 407; Miller, J. A., 169; Mills, C. N., 133, 418; Milne, W. E., 42, 89, 133; Minnick, J. H., 120, 406; Mirick, G. R., 42; Mitchell, H. H., 42, 89, 219, 276; Mitchell, U. G., 105, 180; Mittag-Leffler, 372; Mohr, F., 363; Mohrmann, A., 371; Monroe, W. S., 360; Montague, Josephine, 208, 209; Montell, P., 223; Moore, Charlotte, 168; Moore, C. L. E., 120; Moore, C. N., 90, 93, 162, 163, 234, 280, 405; Moore, E. C., 372; Moore, E. H., 42, 88, 93, 106, 280; Moore, R. A., 179; Moore, R. E., 179; Moore, R. L., 161, 163, 258, 276, 362, 407; Moots, E. E., 322; Morgan, F. M., 88, 117, 405, 418; Moritz, R. E., 160, 261; Morley, F., 90, 133, 224, 255, 324; Morris, C. C., 133, 308; Morse, H. C. M., 43, 275; Moulton, E. J., 93, 105, 221, 364, 417, 420; Moulton, F. R., 43, 89, 180, 219, 276, 277, 324; Moynihan, H., 436; Muir, T., 408; Mullins, G. W., 223, 275, 308; Musselman, J. R., 43; Myers, G. W., 121, 180.

Naebauer, M., 159; Nassau, J. J., 43; Naylor, Mrs. D., 84; Nellis, Evangeline, 309; Nelson, Agnes, 309; Nelson, A. L., 257, 277; Nelson, A. N., 168; Nelson, C. A., 275; Nelson, I. I., 43, 221; Neovius, E. R., 86; Nernst, W., 403; Nicholson, J. W., 220; Nicolet, J., 104; Niemann, Christine, 409; Nipher, F. E., 163, 208, 306; Noether, M., 86; Nörlund, N. E., 323; Normand, E., 364; Norris, Eleanor, 365; Norton, A. H., 43, 84; Nowlan, F. S., 178; Noyes, A. A., 88; Nyswander, J. A., 43.

Oglesby, E. J., 43, 87, 121, 222, 261; Olmstead, Eleanor, 364; Olney, A. C., 439; Olson, H. L., 461; Osgood, W. F., 162, 222, 224, 262, 303, 361; Ott, E., 179; Ott, L. F., 238; Owens, —., 43; Owens, F. W., 221; Owens, Helen B., 84, 222.

Paddock, C. E., 406; Paine, C. P., 87, 180; Papps, P. C. H., 278; Parker, W. P., 104; Parshall, L. R., 237; Parsons, F. W., 238, 418; Pastor, J. R., 302; Pearl, Esther, 409; Peckham, Anna B., 238, 308; Pegram, Annie McK., 387; Peirce, B., 405; Peirce, B. O., 262, 263; Peirce, J. M., 262, 263; Pell, A., 120; Pell, Anna J., 208; Pell, Gladys, 168; Pérès, J., 371; Perry, Winona, 167; Pettitt, H. P., 134, 279, 309; Pfaltz, Mimosa, 308; Pfeiffer, G. A., 119, 178; Phahn, H. R., 322; Phillips, H. B., 85, 120; Piazetti, P., 90; Pickering, E. C., 134; Pierce, T. A., 43; Piersall, Catherine, 308; Pitcher, A. D., 162; Pitman, J. H., 168; Pocock, R. J., 135; Pooler, L. G., 104; Poor, C. L., 116, 121; Poor, V. C., 276; Porter, Flora, 43; Porter, H. E., 43; Porter, M. B., 161, 222, 364; Porter, P. C., 43; Posey, F. D., 43; Post, E. L., 223, 258; Powell, H. W., 84; Powell, Margaret E., 169; Prante, Beulah, 309; Pratt, Gertrude, 409; Pride, H. H., 43; Priest, Almira K., 409; Pritchard, R. V., 238; Pritchett, H. S., 419; Putnam, T. M., 133, 418.

Quick, Hazel, 209; Quigley, Jessie G., 438.

Rafferty, P., 238; Raiford, T. E., 462; Ramsey, A., 85; Randolph, O. A., 237; Rankin, W. W., 418; Ransom, W. R., 43, 67, 86, 277; Rayleigh, see Strutt, J. W.; Raymond, Ellen, 168; Reed, F. W., 43, 87, 134; Reed, L. J., 43; Rees, E. L., 279, 309; Reeve, W. D., 67, 435; Rehm, H. W., 221; Reichel, O., 86; Requa, Emma M., 168; Reye, T., 323; Reynolds, C. N., 420; Rhoades, A. L., 43; Rhoton, A. L., 134; Rice, C. D., 307, 404; Rice, H., 104; Rice, J. N., 43; Rich, F. M., 164; Richardson, C. H., 84, 104, 279; Richardson, Flavia L., 263; Richardson, R. G. D., 106, 167, 383; Rider, P. R., 43, 165; Rietz, H. L., 97, 99, 106, 133, 359; Ritt, J. F., 43, 223, 405; Robbins, R. B., 277, 378; Robinson, Opal M., 168; Robison, G. M., 370; Roe, E. D., Jr., 120, 323; Roever, W. H., 43, 67, 89, 102, 106; Rogers, Agnes L., 406; Rogers, J. C., 238; Roman, I., 43, 87, 219, 322; Root, R. E., 325; Rosenbach, J., 87; Rosenthal, Anita, 168; Rothrock, D. A., 86, 106; Rouse, L. J., 277; Routh, E. J., 220; Rowe, J. E., 43; Roy, L., 371; Rupp, C. A., 275; Rusk, W. J., 67; Russell, B., 160; Russell, H. N., 121.

Sage, J. R., 387; Sallade, J. A., 43; Salmon, G., 160; Sanden, H. v., 159; Sanders, S. T., 122; Sanford, S. P., 43; Sarton, G., 118–119, 121, 207, 305, 361; Satterly, J., 169; Savage, Georgie, 364; Schaaf, Georgia, 308; Schick, Minna J., 275; Schlauch, W. S., 223; Schlesinger, F., 375; Schlesinger, L., 161; Schmidt, A., 160; Schmiedel, O., 306; Schoenflies, A., 403; Schorling, R., 67, 280, 439, 463; Schreiber, E. W., 306; Schweidler, E. v., 160; Searle, G. M., 85; Seely, Caroline E., 43, 258; Segre, C., 160; Selves, Mrs. Catharine, 387; Sensenig, W., 257; Severini, C., 371; Shafer, Vesta, 209; Shamos, M., 238; Sharp, Thelma, 409; Sharpe, F. R., 221, 274; Shaver, W. W., 169; Shaw, J. B., 383; Shepard, E. I., 43; Sherk, W. H., 461; Sherwood, G. E. F., 372; Shively, L. S., 461; Shook, C. A., 43; Short, C. A., 43; Showman, H. M., 83, 322; Shumaker, A., 308, 309; Siceloff, L. P., 43, 223, 262; Silverman, L. L., 43, 417, 462; Simonds, W., 364; Simpson, T. McN., 43, 84, 462; Singer, C., 119; Sisam, C. H., 207, 237, 257, 404; Skinner, A. N., 85; Skinner, C. H., 164; Slaught, H. E., 92, 93, 105, 106, 180, 220, 224, 374; Slichter, C. S., 67, 180; Slobin, H. L., 43, 85, 322; Slocum, S. E., 235; Smail, L. L., 120; Smedley, Caroline H., 169; Smith, C. E., 43; Smith, Clara E., 365; Smith, C. M., 134; Smith, D. E., 85, 90, 93, 105, 106, 258, 277, 280, 380, 439; Smith, E., 409; Smith, E. R., 406; Smith, E. S., 43, 180; Smith, G. W., 85, 257, 279, 309; Smith, H. B., 387; Smith H. L., 43, 84, 322, 462; Smith, J. B., 133; Smith,

Margarethe M., 238; Smith, M. G., 87, 322; Smith, O. A., 204; Snapp, R. R., 221; Snedden, D., 360; Snyder, V., 90, 221, 277, 420; Somers, R. H., 43; Sommerfield, A., 168; Sommerville, D. M. Y., 161; Sonley, A. J., 169; Spaulding, E. G., 220; Sperry, Pauline, 133; Staeckel, P., 161; Stager, H. W., 119; Stanwick, C. A., 43; Stark, Marion E., 219; Staude, O., 85; Stearns, C. L., 86; Stebbins, J., 374; Stephanos, C., 86; Stephens, E., 238; Stephens, H. N., 418; Stephens, Ruth, 209; Stetson, J. M., 43, 370; Stockard, L. V., 261; Stokes, C. N., 387; Stoll, Laura, 309; Stone, R. B., 104; Stouffer, E. B., 180, 208; Stradler, W. W., 164; Stratton, S. W., 324; Stromquist, C. E., 120; Stromsky, S., 405; Street, G. T., 43; Strunsky, S., 405; Strutt, J. W., 323; Sullivan, C. T., 408; Swann, W. F., 436; Swartzel, K. D., 363; Sweet, E. L., 167; Sylow, L., 119, 164.

Tait, P. G., 178; Tanner, J. H., 90, 136, 238; Tanzola, J. J., 44; Taylor, A. E., 455; Taylor, E., 370, 419; Taylor, J. S., 275; Terrill, H. M., 44; Thayer, G., 44; Thompson, R., 308; Thorne, P. L., 44, 179; Thorp, Ella, 209; Thulin, F., 303; Tileston, R. R., 387; Timerding, H. E., 159; Todhunter, I., 220; Tolman, E. C., 303; Torek, Gretchen, 308; Torroja, D. E., 302, 323; Touton, F. C., 359, 419, 420; Tremaine, D'Estell, 209; Trimmer, J., 168, 169; Tripp, M. O., 164, 258, 263, 419; Trorer, Elizabeth, 169; Tsoo, C., 420; Tucker, R. S., 275; Turner, R. L., 44; Tuttle, F. W., 310; Twigg, H. J. R., 163; Tyler, H. W., 44, 93, 280; Tyler J., 219.

Underhill, A. L., 179; Underwood, P. H., 439; Underwood, R. S., 462; Upton, C. B., 168, 223.

Valiron, G., 371; Van der Vries, J. N., 44, 106, 178; Vance, E. H., 462; Vandiver, H. S., 44, 162, 370, 405; Vann, May H., 221; Van Vleck, E. B., 322; Vaughan, W. S., 169; Veblen, O., 44, 89, 219, 223; Véronnet, A., 371; Veronese, G., 179; Vessist, E. P. J., 371; Villat, H., 371; Volterra, V., 324, 404.

Waddell, S., 164; Walker, Margaret, 309, 363, 364; Wallace, H. G. S., 457; Walsh, C. B., 168; Walsh, J. L., 44; Walter, A., 387; Wapple, A. R., 44; Warren, L. A. H., 83; Washington, H. S., 220; Watkeys, C. W., 104; Watkins, Mildred, 261; Watson, Mattie R., 261; Watt, Evelina, 209; Wear, L. E., 303; Weaver, Esther, 308, 309; Weaver, J. H., 404; Weaver, W., 44, 87, 218, 417; Webb, H. E., 278, 383, 384; Webber, W. P., 406; Webster, A. G., 44, 223, 261; Webster, Louisa M., 44, 420; Wechsler, A. L., 44; Wedderburn, J. H. M., 44, 462; Weeks, Eula A., 439; Weinstein, M. B., 179; Weiss, P., 178; Wells, V. H., 44; Wells, W., 359; Werner, Miriam, 168; Wester, R. A., 44; Wheeler, J. J., 180; Whelan, Loretta, 365; Whelan, Marie, 365; Whisenhunt, A. P., 238; Whitcher, Edith, 208; White, C. E., 164; White, H. S., 277, 304, 405, 419; White, Marion B., 178; White, P. J., 264; Whitehead, A. N., 455; Whitford, D. B., 167; Whitney, Fern, 308; Whittemore, J. K., 122, 162, 258; Wiechert, E., 178; Wiener, L., 361; Wiener, N., 44, 275; Wilczynski, E. J., 121, 180, 277, 361; Wilder, C. E., 44, 87, 88, 133, 219; Wildman, —, 308; Wiley, F. B., 308; Willard, H. R., 44; Williams, F. B., 44, 85; Williams, Gladys, 364; Williams, K. P., 44; Winger, R. M., 84; Willson, R. W., 323; Wilson, D. T., 44, 85, 235; Wilson, E. B., 223, 277; Wilson, N. R., 44; Wilson, R. E., 93; Wischmeyer, C., 67; Wolf, C., 134; Wolfe, H. E., 44, 88, 461; Wolff, H. C., 180, 461; Wood, F., 44, 417; Wood, F. E., 44, 88, 219, 274; Woodrow, J. W., 237; Woods, B. M., 44; Wright, Frances W., 167; Wright, W. H., 44.

Yaeger, L. E., 84; Yanney, B. F., 384; Yardley, H., 169; Yeaton, C. H., 44, 83, 87, 209, 275, 438; Young, Jessica M., 87; Young, J. W., 88, 90, 93, 106, 117, 134, 255, 277, 279, 358, 384, 417, 463; Young, J. W. A., 180, 361, 403; Young, Nellie, 209; Young, W. H., 462.

Zacharias, M., 160; Zeldin, S. D., 323; Zeuthen, H. G., 323, 404; Ziwet, A., 106, 223.

UNIVERSITIES, COLLEGES, ASSOCIATIONS, ETC.

American Mathematical Society, April meeting, 278; summer meeting, 420; winter meeting, 90; Chicago Section, 132, 223; Committee on the Maxime Bôcher Memorial, 136.

Cambridge University, 220.

Geophysical Institute, England, 324.

Indian Mathematical Society, 135.

Massachusetts Institute of Technology, 323.

Mathematical Association of America, Annual register, 224; Supplement to November MONTHLY; Finance Committee, 224; Summer meeting, 224, 278.

Norwegian Mathematical Society, 324, 455.

Prizes: Academy of Sciences of the Institute of France (six prizes), 223; Adams Prize, 220; Querini-Stampalia Prize, 419, 462.

School of Navigation, Canada, 132.

Stevens Institute of Technology, 87.

Student Army Training Corps, 88.

Summer courses, 133, 180, 221–223, 224, 276–277.

United States Naval Academy, 221, 275–276.

University of California, Southern Branch, 372.

University of Strassburg, 371.

SUPPLEMENT.

Register of officers and members, supplement to the MONTHLY for November, 40 pages, with cover.

2030

INDEX TO VOLUME XXVII, 1920.

Misprinted names in the text are corrected, and missing initials supplied, in this index.

FORMAL PAPERS—AUTHORS.

ADAMS, O. S. The December meeting of the Maryland-Virginia-District of Columbia Section, 147.

——— The May meeting of the Maryland-Virginia-District of Columbia Section, 343–345.

ALTSHILLER-COURT, N. On the orthocentric quadrilateral, 199–202.

ARCHIBALD, R. C. Gauss and the regular polygon of seventeen sides, 323–326.

ARMSTRONG, G. N. Fifth annual meeting of the Ohio Section, 287–289.

BALLANTINE, J. P. A graphic solution of the cubic equation, 203–204.

BENNETT, A. A. Modular geometry, 357–361.

BRADLEY, H. C. The graphical solution of spherical triangles, 452–459.

CAIRNS, W. D. Fourth annual meeting of the Mathematical Association of America, 93–112.

——— Mathematical Association of America, Council of, 243–244.

——— Fifth summer meeting of the Association, 385–393.

CANDY, A. L. A mechanism for the solution of an equation of the nth degree, 195–199.

——— Elimination of skidding due to steering mechanism on motor cars, 245–252.

COFFIN, L. M. The April meeting of the Iowa Section, 346.

CRESSE, G. H. Expressions for certain accelerations of a particle, 402–404.

DUNKEL, O. Note on Caustics, 225–226.

ECHOLS, W. H. Note on the roots of the derivative of a polynomial, 299–300.

JORDAN, H. E. The April meeting of the Kansas Section, 283–285.

LIGHT, G. H. The April meeting of the Rocky Mountain Section, 289–290.

LYTLE, E. B. The November meeting of the Illinois Section, 112–113.

——— The college as a training school for high school teachers, 157–163.

MATHESON, J. Continuity in synthetic geometry, 47–53.

MERRILL, A. S. The "danger area" curve, 398–401.

METZLER, W. H. Some vanishing aggregates connected with circulants, 11–14.

MILLER, N. A graphical aid in the study of functions of a complex variable, 354–357.

MORLEY, F. V. Note on the incenters of a quadrilateral, 252–255.

PAPPS, P. C. H. Mathematics and life insurance, 291–299.

RIDER, P. R. The third annual meeting of the Missouri Section, 148–151.

RIGGE, W. F. Envelope rosettes, 151–157.

SLAUGHT, H. E. Retrospect and prospect for mathematics in America, Presidential address, 443–451.

SMITH, G. W. The April meeting of the Kentucky Section, 285–286.

SMITH, L. L. A construction of the regular polygon of seventeen sides, 322–323.

WEAVER, J. H. Some extensions of the work of Pappus and Steiner on tangent circles, 2–11.

WEAVER, W. The average reading vocabulary; an application of Bayes's theorem, 347–354.

WESTER, C. W. The April meeting of the Iowa Section, 1.

FORMAL PAPERS—TITLES.

Accelerations of a particle, Expressions for certain. G. H. CRESSE, 402–404.

Aggregates connected with circulants, Some vanishing. W. H. METZLER, 11–14.

Annual meeting, Fourth, of the Mathematical Association of America. W. D. CAIRNS, 93–112.

Bayes's theorem, The average reading vocabulary; an application of. W. WEAVER, 347–354.

Caustics, Note on. O. DUNKEL, 225–226.

Circles, Tangent, Some extensions of the work of Pappus and Steiner on. J. H. WEAVER, 2–11.

Circulants, Some vanishing aggregates connected with. W. H. METZLER, 11–14.

Complex variable, Functions of a, A graphical aid in the study of. N. MILLER, 354–357.

Construction of the regular polygon of seventeen sides. L. L. SMITH, 322–323.

Continuity in synthetic geometry. J. MATHESON, 47–53.

Curve, The "danger area." A. S. MERRILL, 398–401.

Derivative of a polynomial, Note on the roots of the. W. H. ECHOLS, 299–300.

Election of members. W. D. CAIRNS, 243–244.

Envelope rosettes. W. F. RIGGE, 151–157.